Abstract: The present paper proposes a synchronization method of a neural oscillator to a meshing frequency of the vibration signal from a gear pair. The proposed method could enable the angular position of the meshing gear to be estimated without any help of speed sensors. The neural oscillator is designed to tune its natural frequency to the meshing frequency of a gear pair, and the acceleration responses, which are measured at the top of the bearing housing, are input to the neural oscillator. Although, the vibration caused by the tooth-to-tooth meshing of a rotating gear pair contains various frequency components, the properly-designed neural oscillator can follow the change in meshing frequency due to the driving torque variation autonomously, because of the synchronization property of the oscillator. The proposed method was applied to the experimental acceleration response in operating tests on POM (Polyoxymethylene) gears. As a result of the application, the meshing periods of the vibration signal was extracted.

Keywords: Meshing vibration, Detection of phase angle, Neural oscillator, Synchronization

1 Introduction
Trend toward an increase in plastic gear applications has continued, because of cost effectiveness, low density, capability to absorb vibration, ability to operate with no lubricant, and so on. If the limitation of plastic gears relative to metal gears would be improved as a result of new developments in both materials and processing, plastic gears could be used under more severe conditions such as high load and/or high rotation speed. Even before such developments appear, if an emergency system shutdown could be provided in a safe manner, applications such as high load and/or high rotation speed. Even before plastic gears could be used under more severe conditions new developments in both materials and processing, relative to metal gears would be improved as a result of lubricant, and so on. If the limitation of plastic gears capability to absorb vibration, ability to operate with no such developments appear, if an emergency system torqu variation autonomously, because of the effects of gear tooth cracks at its tooth root on measured acceleration responses. The acceleration responses were measured with a pickup set at the top of a housing of the driven-side gear shaft bearing during the operating tests, and the frequency analyses were carried out to identify dominant frequency components in the tests. As a result of the frequency analyses, the responses included not only the DC component, shaft frequency, its harmonics, fundamental meshing frequency, some mesh harmonics, and its modulating sidebands, but also rolling-elements noise of bearings, motor vibration and its harmonics, and so on. The particularly conspicuous frequency components in the responses were the shaft rotation frequency, fundamental meshing frequency, some mesh harmonics, and its modulating sidebands, whose frequency is identical with the shaft frequency, and the responses were strongly affected by amplitude and frequency modulations. These modulation phenomena may arise from the facts that plastic gears are so flexible because their tooth stiffness is lower than that of metal gears, and are subject to greater dimensional instabilities due to their larger coefficient of thermal expansion. These modulating components complicate the procedure of frequency analyses of the acceleration responses and hide plastic-gear-failure signs. For example, in the frequency analyses of the response data, the peak of the meshing frequency results in a slight change in complete tooth fracture [1]. However, the response data including amplitude, frequency and phase modulation, rolling-elements noise of bearings and motor, driving torque variation, and so on, cause detection of gear failure signs to be difficult in the frequency analyses, because the detection requires slight changes in the complex data to be distinguished.

In order to avoid the difficulties in the frequency analyses, the time synchronous averaging (TSA) of the acceleration responses is one of the most powerful tools to cancel vibration incoherent with the revolution period of the objective gear in time domain [2]. However, the TSA method generally needs the measurement of the rotation speed of one gear of interest, which is usually delivered by a tachometer or encoder. The attachment of these sensors is possible for limited conditions only in some particular environments; therefore, a health monitoring system of gears without any speed sensors would suit for engineering applications.
On the other hand, nonlinear oscillators, which were models for rhythm generators consisting of neurons and called “neural oscillators", have been studied in a biological study [3]. Neural oscillators have a characteristic of synchronizing with periodic external inputs in a certain frequency range. Mathematical models of the neural oscillators were also proposed and examined [4][5], and its applications to walking robot were reported [6][7]. Iba and Hongu studied a new control system for active mass dampers using the mathematical model of the neural oscillators, which can follow the vibration behaviour of high-rise buildings due to the synchronization characteristic [8][9]. These studies have suggested the possibility the neural oscillator being used as an estimator of the rotation speed of the gear of interest without any speed sensors. If the neural oscillator was synchronized with the vibration signal of the gear meshing, the oscillator would enable vibration components within the particular frequency range to be extracted from noisy signals and the rotation speed of gears to be estimated autonomously, following the designated frequency even with some driving torque variations.

The present paper proposes a new signal processing method, in which the neural oscillator synchronizes to a meshing frequency for estimation the angular position of the meshing gear pair without the help of speed sensors. The basic concept of the proposed system is first mentioned, then, the neural oscillator that constructs the proposed system and the design method of neural oscillator is explained. After that, the proposed method is applied to the experimental acceleration response of operating tests on POM (Polyoxymethylene) gears to confirm the validity of the proposed method, and the results are examined.

The ultimate goal is to estimate the angular position of the meshing gear pair without speed sensors and to carry out a time synchronous average method to the measured response for detection the failure signs of gears, but the design method of the synchronization system is mainly discussed in this paper, so that there is no direct mention of the failure detection method.

2 Neural Oscillator
In this section, the mathematical model of a neural oscillator is introduced, and the synchronization property of the oscillator, which is one of the considerable properties of nonlinear oscillators, is explained.

2.1 Mathematical model
In this study, Matsuoka's neural oscillator [4][5] is used as the neural oscillator. The mathematical model of Matsuoka’s neural oscillator is expressed as follow.

$$X(t) = AX(t) + B \max(0, X(t)) + S + E p(\Omega t)$$

(1)

Where, the variable $X(t)$ is composed of four variables and is the vector.

$$X(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_1'(t) & x_2'(t) \end{bmatrix}$$

Moreover, the output of the each oscillator is obtained as follow.

$$y(t) = C \max(0, X(t))$$

(2)

Additionally, $p(\Omega t)$ is a periodic external signal, whose frequency is $\Omega$. The components of the matrices and vectors are obtained as

$$A = \begin{bmatrix} -1/\tau & 0 & -b/\tau & 0 \\ 0 & -1/\tau & 0 & -b/\tau \\ 0 & 0 & -1/T & 0 \\ 0 & 0 & 0 & -1/T \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -a/\tau & 0 & 0 \\ -a/\tau & 0 & 0 & 0 \\ 1/T & 0 & 0 & 0 \\ 0 & 1/T & 0 & 0 \end{bmatrix}, \quad S = \begin{bmatrix} s/\tau \\ s/\tau \\ 0 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} \epsilon \\ -\epsilon \end{bmatrix}$$

$$C = [1 \ -1 \ 0 \ 0]$$

This model contains two first order lag elements to express excitation and inhibition, and can generate sustained oscillation, even if input gain $\epsilon$ is zero and there is no external input. Generally, the five coefficients $s$, $r$, $T$, $b$, $a$, have been decided by identification of a biological neuron. But the specified animate being is not considered in this study. These parameters are chosen by the following design method to generate oscillation with a desired natural frequency $\omega$ and output amplitude $A_{\text{max}}$.

$$\begin{align*}
\tau &= \frac{0.212}{\omega} \\
T &= 12\tau \\
a &= \frac{(T - \tau)^2}{4T\tau} \\
b &= a \\
s &= A_{\text{max}} \times 0.612
\end{align*}$$

(3)

According to the mathematical model of the neural oscillator, a numerical simulation is carried out using Runge-Kutta 4th order method. In this simulation, the desired natural frequency of the oscillator is 1 [rad/s], and the external signal is eliminated from the equation (1). The desired amplitude of the neural oscillator is 1 in this simulation. Figure 1 shows the time history of the autonomous oscillation. The parameters of neural oscillator are shown in Table 1.
### Table 1. Parameters of Matsuoka’s neural oscillator

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>1.634</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.212</td>
</tr>
<tr>
<td>$T$</td>
<td>2.54</td>
</tr>
<tr>
<td>$b$</td>
<td>2.52</td>
</tr>
<tr>
<td>$a$</td>
<td>2.52</td>
</tr>
</tbody>
</table>

#### 2.2 Synchronization region of neural oscillator

In order to estimate the rotation speed of gear without speed sensors, the neural oscillator must be synchronized with the vibration signal of the gear meshing. The synchronization region, where the oscillator can follow the variation of the designated meshing frequency, is one of the important things to understand, and introduced in this subsection.

Here, $p(\Omega t)$ is assumed as a normalized sinusoidal signal in equation (1). The frequency of the periodic external input is $\Omega$. If the frequency $\Omega$ is partially close to the natural frequency $\omega$ of the neural oscillator, the oscillator is synchronized with the external sinusoidal wave. In other words, this synchronization phenomenon can be seen when the detuning $\Omega - \omega$ between the oscillator's frequency $\omega$ and the external periodic force's frequency $\Omega$ is in a range. According to an analysis by the phase reduction method \[10\][11][12][13], the synchronization region $\epsilon_{\min} < \Omega - \omega < \epsilon_{\max}$ can be decided. Here, $\Gamma_{\min,max}$ is the phase coupling function of the oscillator.

Figure 2 shows a synchronization region of the neural oscillator, whose parameters are shown in Table 1. The green-colored area is the region obtained by numerical simulation, which is also called as "Arnold tongue". The synchronization region obtained by the phase reduction method is shown as black dotted lines in the figure.

![Fig. 2. Synchronization region of neural oscillator](image)

In this figure, the horizontal axis is the frequency $\Omega$ of the sinusoidal input, and the vertical axis is the amplitude $\epsilon$ of input. It is clear that the synchronization region has spread from the oscillator’s frequency $\omega = 1$ [rad/s], when the amplitude of the external signal is increased. In other words, the synchronization region depends on the amplitude of the input signal. If the coefficient for the external vibration in the neural oscillator is increased or decreased, the region can be freely adjusted.

The neural oscillator has a sensitive reaction to the sinusoidal input within the synchronization region, however, not has the sensitivity to the input outside of the region. Therefore, the neural oscillator can be used as an band pass filter to eliminate the unnecessary frequency component in the vibration response of gear systems and enable the meshing frequency component within the particular frequency range to be extracted from the noisy signals, autonomously.

### 3 Definition of Phase

In this section, the phase of the neural oscillator is introduced for estimation of angular position of gears. After this introduction, the phase of gears are discussed.

#### 3.1 Phase of neural oscillator

Assume that the neural oscillator, which is expressed in equation (1), have a stable periodic trajectory $\mathbf{q}(t) = \mathbf{q}(t + T_\text{rot})$ starting from $\mathbf{q}_0$ with period $T_\text{rot}$ and frequency $\Omega = 2\pi / T_\text{rot}$ under synchronization. The parameterization $\theta(t)$ of by a phase $\theta$ is illustrated in this subsection. Where $\mathbf{q}_i$ is assumed to have zero phase. In practice, the choice of $\mathbf{q}_i$ is arbitrary, but the zero phase point is defined to be $y = 0$, $\dot{y} > 0$ in this study. Here, in order to assign a phase value to each point on the periodic trajectory, $\theta(t)$ is defined as a solution to the next phase equation.

$$\theta(t) = \Omega t, \quad 0 \leq t \leq T_\text{rot}$$

(4)

Although the neural oscillator makes a rotation on the periodic trajectory with possibly a non-constant speed, the phase can makes a rotation with a constant speed $\Omega = 2\pi / T_\text{rot}$.

#### 3.2 Angular position of gear

While the neural oscillator is synchronized with the vibration signal of the gear meshing, the oscillator can follow the variation of the designated frequency, autonomously. In other words, the period of the vibration signal of the gear meshing is sequentially updated during a rotation of the gear, if the external periodic frequency $\Omega$, or the meshing frequency, is in the synchronization range. The updated period, frequency and phase of the neural oscillator are expressed as follow.

$$T_{\text{rot},i} = t \cdot N_{\text{tooth}}$$

$$\Omega_{\text{rot},i} = \dot{\theta} \cdot N_{\text{tooth}}$$

$$\theta_i = \frac{1}{T_{\text{rot},i}} \int_0^{T_{\text{rot},i}} \theta_i \, dt$$

(5)

where, $N_{\text{tooth}}$ is the number of gear tooth of interest. The discrete angular position of the gear is then obtained by:

$$\phi_{\text{gear}} = \frac{1}{N_{\text{tooth}}} \sum_{i=1}^{N_{\text{tooth}}} \theta_i$$

(6)

According to this method, the angular position of the gear can be identified, if the neural oscillator is synchronized with the meshing frequency of the gear systems to extract the meshing period.
4 Experimental Detail and Results

In this section, the proposed estimation method of the gear’s angular position using the neural oscillator is applied to the actual vibration signal of rotation fatigue tests of POM (Polyoxymethylene) gears, and the instantaneous meshing periods are identified.

4.1 Test gear and power-absorb-type gear test rig

In this study, the analysis object by the proposed method is a POM spur gear. The driven gear is shown in Fig. 3. The module of the gear is 1.0 [mm], and the number of teeth is 48. The material of the driving gear is steel, and the number of teeth is 67.

![Fig. 3. Test gear (Polyoxymethylene, Module =1.0, Number of teeth = 48)](image)

Next, the power-absorption-type gear operating test rig for operating test of POM gears is shown in Fig. 4. ① is the driving motor, and ② is the driven motor to absorb the power of the driving motor. ⑥ is the deriving gear driven by V type belt ②. In addition, the torque and rotation speed of test gear ⑤ is controlled by variable resistances. The acceleration responses of the test rig were measured with a pickup set ⑥ at the top of a housing of the driven-side gear shaft bearing during the operation tests. In addition, ②, ③, ⑦ and ⑨ are the driving belt, adjusting table for gap, torque sensor and microphone.

4.2 Vibration signal from gear system

One example of the acceleration responses for 0.01 [s] measured at the top of a housing of the driven-side gear shaft bearing is shown in Fig. 5. The rotation speed of the shaft is about 3000 [min⁻¹], or 50 [Hz], and the meshing frequency is about 2400 [Hz]. The driving torque measured by the torque sensor is about 7.5 [Nm]. Time range of this figure is about one-half turn of gear of interest, and it appear to have 24 cyclic peaks, but it is not clear due to effect of amplitude, frequency and phase modulation on the data. Figure 6 shows the frequency spectrum of the measured acceleration for 10 seconds. Especially, this figure is an enlarged view around 2400 [Hz], which is the meshing frequency of the gear pair.

![Fig. 5 Acceleration response of gear system](image)

![Fig. 6 Frequency spectrum of acceleration response](image)

As can be seen, the acceleration response includes not only the fundamental meshing frequency, but also its modulating sidebands affected by the shaft frequency [14][15][16]. In addition, the responses has the DC, shaft frequency, its harmonics, rolling-elements noise of bearings, motor vibration and its harmonics, and so on, such complex frequency components cause detection of gear failure signs to be difficult.

4.3 Extraction of meshing period by synchronization

In order to estimate the angular position of gear for the time synchronous average without speed sensors, the meshing period of the gear system must be identified by the neural oscillator synchronized with the gear meshing of vibration signal. In this subsection, the proposed synchronization method is applied to the measured acceleration data. According to equations (2), the parameters of Matsuoka neural oscillator were designed.

Figure 7 shows the time history of the input acceleration to the neural oscillator and the output from the neural oscillator. The acceleration response after a lapse of 5 minutes is used as the acceleration input to the oscillator. The desired amplitude of the neural oscillator is set to 0.01 and the input gain is 5. In this figure, the output
amplitude of neural oscillator is scaled 2000 times larger than original oscillation for comparison. Figure 8 and 9 shows the close up of the Fig. 7.

As can be seen, the oscillator’s output is simplified, because the complex acceleration response with multiple frequencies except around the meshing frequency is filtered out in the output. Especially, it is obvious that the effect of high harmonic of meshing frequency is reduced in the output, because these harmonics are not in the synchronization region of the oscillator. As a result of synchronization, it is easy to identify the period of the meshing vibration by using the output of the oscillator.

Next, the periods of the neural oscillator, or meshing periods, are extracted from the output of the oscillator. The zero phase point of the neural oscillator was defined to be $y = 0$, $\dot{y} > 0$ in this study. It is difficult to distinguish the difference of the period of the oscillator at a glance, but the period is not constant. According to the definition of the zero phase point, the period of two seconds data was calculated. The result of calculation is shown in Fig. 10. Figure 10 is the histogram of the calculated period for two seconds. The average of the period is $4.1749 \times 10^{-4}$ [s], therefore, the average frequency is 2395.3 [Hz]. The variance of the period is $2.5231 \times 10^{-10}$. These values are depended on the synchronization region, which is controlled by the input gain $\epsilon$ of the neural oscillator. In other words, the input gain should be defined as a properly-designed value. However, in this experiment, the rotation speed of the gear was not measured and there was no true value. The synchronization of the oscillator was observed and the period of the oscillator was calculated, but the evaluations of the proposed system have not been conducted, yet.

5 Conclusions

An application method of a neural oscillator was proposed to extract the instantaneous meshing periods of the vibration signal from gear systems without speed sensors. This method can be used as fundamental signal processing for estimation of the gear angular position without the help of speed sensors. First of all, the design method of the neural oscillator, which has a synchronization property, was showed. The neural oscillator was designed to tune the natural frequency to the meshing frequency of the gear system. As a result, the properly-designed neural oscillator was able to follow the change in meshing frequency. Additionally, the oscillator showed a function similar to a band-pass filter, therefore, the frequency components, which are not in the synchronization region, can be filtered out. Because the filtered output consists of easy-to-use signals, the identification of the oscillator’s period is technically feasible. The identified periods of the oscillator indicate the meshing periods, and the angular position of the gear can be calculated by the meshing period. In this paper, the proposed method was applied to the experimental acceleration response of operating tests on POM (Polyoxymethylene) gears, and the synchronization of the proposed system was confirmed.
and the period of the meshing was calculated. As a result, it was confirmed that the proposed system had reasonably effective for practical use. However there was no true value of the rotation speed of the gear in the operating tests, the evaluation of the proposed system have not been conducted and the design method of the input gain $\epsilon$ of the oscillator has not been discussed in this paper. One of the future works is to measure the true rotation speed and vibration of the meshing gear system, and to evaluate the estimated values.

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References