Reconfigurable Parallel Mechanisms with Three Types of Kinematotropic Chains

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Abstract: This paper presents the idea of constructing reconfigurable kinematic limbs by integrating kinematotropic linkages as subchains. Three kinematotropic chains characterized by change in pair connectivities without change of the number of degrees of freedom are concerned and the branch transitions are analyzed. Reconfigurable kinematic limb that provides a constraint couple in one configuration and a constraint force in the other configuration is desired and the constraint of serial chain used in the construction of this kind of limb is identified. The requirements of constructing reconfigurable limbs with three kinematotropic chains are analyzed. A class of reconfigurable parallel mechanisms is constructed by connecting a moving platform to a fixed base with three identical reconfigurable limbs. The platform of each reconfigurable parallel mechanism has the ability to perform various motion modes including 3T motion, 2T1R motion, 2R1T motion and 3R motion. Two reconfigurable parallel mechanisms are sketched as examples and the actuation scheme for the mechanisms in this class is discussed.

Keywords: Reconfigurable kinematic limb, Kinematotropic chain, Reconfigurable parallel mechanism, Actuation scheme

I. Introduction

Since the 1990s, researchers have become interested in the study of reconfigurable mechanisms and robots with the ability of reconfiguration and mobility change, which are superior to the traditional fixed-DoF mechanisms in some applications with various task requirements. Wohlhart [1] proposed the kinematotropic linkages characterized by variations in the position of variables can result in the permanent finite mobility. Galletti and Fanghella [2] presented four basic single-loop kinematotropic linkages through a systematic approach based on displacement group theory. Subsequently, the study progressed to the method for identifying multi-loop kinematotropic mechanisms that can change their group of motion [3]. Dai et al. [4] proposed the metamorphic mechanisms with changeable mobility resorting to the topological structure variations, followed by several works focused on the characteristic analysis [5], matrix representation [6] and structure synthesis [7] of this kind of mechanisms.

Compared with the traditional parallel mechanisms that possess constant degrees of freedom, reconfigurable parallel mechanisms not only keep the advantages such as high stiffness and good operation accuracy, but also gain the ability of reconfiguration and adaptability. These determine that the reconfigurable parallel mechanisms have significant potential to be put into practical applications. Several novel reconfigurable parallel mechanisms have been proposed in the past decades. Fanghella et al. [8] presented several parallel mechanisms that can change their group of motion. Kong et al. [9] proposed a general method for type synthesis of parallel mechanisms with both pure rotational and pure translational operation modes. Gogu [10], [11] addressed the branching singularities in kinematotropic parallel mechanisms. Li et al. [12] presented a new family of parallel mechanisms with bifurcation of Schönflies motion. Maurizio Ruggiu et al. [13] analyzed the mobility and kinematics of a parallel mechanism with both PPR and planar operation modes. It is obvious that there exist singular configurations in these mechanisms. These configurations are called as constraint singularity and C-space singularity [14], [15] where the reconfigurations together with mobility changes happened.

As an important part of reconfigurable parallel mechanisms, metamorphic parallel mechanisms that can change their topological structures during operation have attracted substantial interest in the research community. Based on some ingenious design of metamorphic joints, several metamorphic parallel mechanisms are constructed in recent years. Zhang et al. [16], [17] proposed a vA joint with three mobility phases and integrated it in the construction of metamorphic parallel mechanisms. Gan et al. [18], [19], [20] presented a new joint coined as rT joint and put forward a general procedure for mobility-change-aimed metamorphic parallel mechanism construction.

However, the aforementioned reconfigurable parallel mechanisms have a common drawback, i.e. some actuators mounted far away from the base are required in these mechanisms for realizing reconfigurations. For example, two revolute joints adjacent to the moving platform must be actuated in the reconfigurable parallel mechanisms presented in [12]. These actuation schemes have negative effects on the mechanisms’ dynamic performance. Therefore, the actuation schemes of reconfigurable mechanisms with parallel structures should be taken into consideration in the design stage.

Since the kinematotropic linkages have different motion characteristics in different branches, it is possible to construct reconfigurable parallel mechanisms by integrating kinematotropic linkages as subchains in the kinematic limbs. In this paper, three kinematotropic chains characterized by change in pair connectivities without change of the number of degrees of freedom are concerned. Reconfigurable limbs are assembled with one of the three kinematotropic chains and a serial chain that can provide a
constraint couple and a constrain force. A class of reconfigurable parallel mechanisms are constructed with three identical reconfigurable limbs connecting the moving platform and the base. The ability of performing various motion modes with the reconfigurable limbs reconfiguring into different configurations is revealed. Actuation scheme for the mechanisms in this class is discussed.

II. Three Kinematotropic Chains

The advantages of parallel mechanism with fewer than six degrees of freedom (low-DoF parallel mechanism) compared with 6-DoF parallel mechanisms such as larger workspace, simpler kinematics and simpler mechanical design have been commonly realized in research community. For a low-DoF parallel mechanism, the constraints applied on the moving platform are exerted by the parallel limbs, pure force and pure couple are two special kinds of constraint. In 2002, Fang et.al [21] enumerated limb structures that can provide a constraint force or a constraint couple based on screw theory, i.e., the F-limb and the C-limb. These limbs are constructed with general revolute joints and prismatic joints. Characteristics of the joints and the geometrical relationship between the joints are invariant, resulting in constraint invariable of those limbs. As aforementioned, kinematotropic linkages have different motion characteristics in different motion branches can be integrated as subchains to obtain limb structures with the ability of reconfiguration and constraint changing. It is urgently desired that the limb constraint can switch between a constraint force and a constraint couple with the kinematotropic chain in different branches, this kind of limbs is called as L^c\_C reconfigurable limb hereafter. With this in mind, the mobility of the limb in different configurations should be invariable, which determines that the DoF of the kinematotropic chain remains constant in different motion branches. This requirement calls for a special type of kinematotropic chains characterized by change in pair connectivities without change of the number of degrees of freedom. To simplify the analysis, the constant number of DoF of the kinematotropic chain is here assumed to be one. Three kinematotropic chains fit these conditions were presented by Galletti and Fanghella[2], i.e. the spatial 4R chain, the RPRP chain and the diamond chain.

A. The Spatial 4R Kinematotropic Chain

Three configurations of the spatial 4R kinematotropic chain are shown in Fig.1. At the singular position in Fig.1(ab), axes of joint A and joint C are collinear, axes of joint B and joint D are collinear, the kinematotropic chain can separate two branches of motion from this position. By rotating joint B, the chain evolves into branch (a) in Fig.1(a) where joints A and C are locked and performs a rotational motion around joint B(D). The chain can be regarded as an equivalent revolute joint denoted by R_{AB} in this branch. While by rotating joint A at the singular position, the chain evolves into branch (b) in Fig.1(b) where joints B and D are locked and performs a rotational motion around joint A(C). The equivalent revolute joint of the kinematotropic chain in branch (b) is denoted by R_{AC}. Two actuators should be mounted to joints A and B to actuate the kinematotropic chain transforming into different motion branches from the singular position. However, only one actuator is needed in both branch (a) and branch (b).

![Fig. 1. The 4R kinematotropic chain](image)

B. The RPRP Kinematotropic Chain

Fig.2 shows three configurations of the RPRP kinematotropic chain. At the singular position in Fig.2(ab), axes of prismatic joint B and joint D are parallel to each other, axes of revolute joint A and joint C are collinear. The kinematotropic chain can separate two branches of motion from this position. By actuating the actuator mounted to prismatic joint B, the chain evolves into branch (a) in Fig.2(a) where joints A and C are locked and performs a pure translational motion. While by rotating joint A at the singular position, the chain evolves into branch (b) in Fig.2(b) where joints B and D are locked and performs a rotational motion around joint A(C), the equivalent revolute joint of the kinematotropic chain in this branch is denoted by R_{AC}. Two actuators mounted to joints A and B are needed for the kinematotropic chain at the singular position but only one is active when the chain is in branch (a) or branch (b).

![Fig. 2. The RPRP kinematotropic chain](image)

C. The Diamond Kinematotropic Chain

The diamond kinematotropic chain in its three configurations are shown in Fig.3. At the singular position in Fig.3(ab), all the four links are coincident and axes of joint A and joint C are collinear. The kinematotropic chain can separate two branches of motion from this position. By rotating joint B, the chain evolves into branch (a) in Fig.3(a) where the chain behaves as a planar four-bar parallelogram.
While by rotating joint $A$ at the singular position, the chain evolves into branch (b) in Fig. 3(b) where joints $B$ and $D$ are locked and performs a rotational motion around joint $A(C)$. The equivalent revolute joint of the kinematotropic chain in this branch is denoted by $R_{AC}$. Two actuators mounted to joints $A$ and $B$ are needed to actuate the chain transforming into different motion branches from the singular position. Only one motor mounted to joint $A$ is used in branch (b) and the chain is redundantly actuated in branch (a) since all the joints are active.

![Fig. 3. The diamond kinematotropic chain](image)

It is easy to know that in branch (a), link $CD$ has an instantaneous translational motion in the direction perpendicular to link $AD(BC)$, the chain is equal to a virtual prismatic joint, definitely the direction of this prismatic joint changes with the movement of line $AD(BC)$. When one of the three kinematotropic chains is integrated as subchain in a kinematic limb, the topological structure of the limb is changeable due to the kinematotropic property of the subchain, which leads the constraint exerted by the limb changeable. As aforementioned, $L^f_{C}$ reconfigurable limb structures that can provide a constraint force in one configuration and a constraint couple in the other configuration are desired, denotes the constraint force as $F$ and the constraint couple as $C$. It is preferable to arrange the kinematotropic chain at the bottom of a kinematic limb to achieve good dynamic performance since two actuators are mounted to each kinematotropic chain, which determines that the reconfigurable limb will be formed by connecting a kinematropic chain and a serial chain with four degrees of freedom, denotes the wrench system of the serial chain as $W$, two formulations are obtained,

$$
\begin{align*}
    C & \subset W \\
    F & \subset W
\end{align*}
\tag{1}
$$

The wrench system of serial chain with four degrees of freedom contains two bases, the following formula is obviously satisfied since $F$ and $C$ are independent with each other.

$$
W = \{F, C\}
\tag{2}
$$

Serial chain structures meet with this condition are denoted by $L_{FC}$ limbs and listed in Table 1, which have been synthesized in our previous literature [22]. $L^f_{C}$ reconfigurable limb structures will be constructed in the following section.

<table>
<thead>
<tr>
<th>Limb type</th>
<th>Three-link limbs</th>
<th>Two-link limbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2R2P</td>
<td>PPR, R, PR, C</td>
<td>PU, PR, C</td>
</tr>
<tr>
<td>3R1P</td>
<td>PR, R, R, U</td>
<td>PR, C, R</td>
</tr>
</tbody>
</table>

**TABLE 1. $L_{FC}$ limb structures.**

In Table 1, $R_1$ denotes a revolute joint whose axis is parallel to the constraint force, $R_2$ denotes a revolute joint whose axis is intersect with the constraint force, $P$ is a prismatic joint with its axis perpendicular to the constraint force. The common normal of $R_1$ and $R_2$ defines the direction of the constraint couple.

### III. $L^f_{C}$ Reconfigurable Limb Structures

To achieve $L^f_{C}$ reconfigurable limb that can provide a constraint force in one configuration and a constraint couple in the other configuration by connecting a kinematotropic chain and a $L_{FC}$ limb, some necessary geometrical conditions should be satisfied when the limb is constructed.

#### A. Constructed with Spatial 4R Kinematropic Chain

If there exist $L^f_{C}$ reconfigurable limb structures constructed by connecting a spatial 4R kinematropic chain and a $L_{FC}$ limb, it is easy to conclude that when the $L^f_{C}$ reconfigurable limb evolves into a F-limb, the spatial 4R kinematropic chain should be equal to a revolute joint whose axis is different with axes of joints $R_1$ and $R_2$. Assume that the spatial 4R kinematropic chain is in branch (b), the equivalent revolute joint $R_{AC}$ in this branch should be intersecting or parallel to the constraint force. For the fourteen serial kinematic limbs containing $R_1$ joint, $R_{AC}$ can only be intersect with the constraint force and connected to $R_2$ directly since axis of $R_{AC}$ is different with axis of $R_1$ which is parallel to the constraint force. For the PPR, R, C, R, PR, C, R, PU, U, limbs with no revolute joint $R_1$, $R_{AC}$ can be intersecting or parallel to the constraint force. If $R_{AC}$ is intersecting with the constraint force, the spatial 4R kinematropic chain should be connected to joint $R_2$ directly to get finite motion. However, if $R_{AC}$ is parallel to the constraint force, the 4R chain should be connected to $P$ joint since the direction of the constraint force is determined by the two prismatic joints. To make sure that the $L^f_{C}$ reconfigurable limb is a C-limb when the spatial 4R kinematropic chain changes to branch (a), the equivalent revolute joint $R_{BD}$ in this branch should be
parallel to axis of nearby joint \( R_2 \). With all these requirements satisfied, eighteen \( L_6^F \) reconfigurable limbs are obtained as listed in Table 2. To avoid confusion, the spatial 4R kinematotropic chain is denoted by FR hereafter.

<table>
<thead>
<tr>
<th>Limb type</th>
<th>Only single DoF joints with composite joints</th>
</tr>
</thead>
<tbody>
<tr>
<td>2R2P-FR</td>
<td>PPR_1R_2-FR</td>
</tr>
<tr>
<td></td>
<td>PPR_2-R_2-FR</td>
</tr>
<tr>
<td></td>
<td>PR_1PR_2-FR</td>
</tr>
<tr>
<td></td>
<td>R_1PPR_2-FR</td>
</tr>
<tr>
<td></td>
<td>R_2R_2PP-FR</td>
</tr>
<tr>
<td>3R1P-FR</td>
<td>PR_1R_1R_2-FR</td>
</tr>
<tr>
<td></td>
<td>R_1PR_1R_2-FR</td>
</tr>
<tr>
<td></td>
<td>R_1R_1PR_2-FR</td>
</tr>
<tr>
<td>4R-FR</td>
<td>R_1R_1R_1R_2-FR</td>
</tr>
</tbody>
</table>

**TABLE II.** \( L_6^F \) reconfigurable limb structures with 4R kinematotropic chain.

Different configurations of the \( R_1PR_1R_2-FR \) and \( R_2R_2PP-FR \) limbs are illustrated in Fig. 4 and Fig. 5, axis of \( R_{BD} \) is parallel to axis of \( R_2 \) in Fig. 4(a), axis of \( R_{AC} \) intersect with axis of \( R_2 \) in Fig. 4(b), axis of \( R_{BD} \) is parallel to the axis of nearby \( R_2 \) in Fig. 5(a), axis of \( R_{AC} \) is perpendicular to the plane formed by the two prismatic joints in Fig. 5(b). The limb configurations in Fig. 4(a) and Fig. 5(a) are C-limbs while limb configurations in Fig. 4(b) and Fig. 5(b) are F-limbs.

**B. Constructed with Spatial RPRP Kinematotropic Chain**

If we want to construct \( L_6^F \) reconfigurable limb structures with the RPRP kinematotropic chain and a serial chain listed in Table 1, the RPRP kinematotropic chain should be in branch (b) and equal to a revolute joint denoted by \( R_{AC} \) when the reconfigurable limb evolves into a F-limb. Furthermore, \( R_{AC} \) can only be intersect with the constraint force and connected to \( R_2 \) if the serial chain contains revolute joint \( R_1 \). For the \( PPR_1R_2 \) and \( PPU_{12} \) limbs only contain revolute joint \( R_2 \), \( R_{AC} \) can be intersecting or parallel to the constraint force, however, if \( R_{AC} \) is parallel to the constraint force, the equivalent prismatic joint of the RPRP kinematotropic chain in branch (a) is dependent with the two prismatic joints in the serial chain, which makes twist system of the reconfigurable limb degenerated. Therefore, \( R_{AC} \) should be intersect with the constraint force and connected to \( R_2 \) directly too for these two limbs. C-limb structure is obtained with the RPRP kinematotropic chain changing to branch (a) since the equivalent prismatic joint of the kinematotropic chain is independent with the four joints of the serial chain in a general configuration. Sixteen \( L_6^F \) reconfigurable limbs meet with these requirements are listed in Table 3. Two configurations of the \( R_1PR_1R_2-RPRP \) reconfigurable limb are illustrated in Fig. 6 in which Fig. 6(a) shows a C-limb while Fig. 6(b) shows a F-limb.

**C. Constructed with Diamond Kinematotropic Chain**

Since both the diamond kinematotropic chain and the RPRP kinematotropic chain are equal to a revolute joint in one branch and to a prismatic joint in the other branch, the

**TABLE III.** \( L_6^F \) reconfigurable limb structures with RPRP kinematotropic chain.
requirements for constructing $L^F_C$ reconfigurable limbs with diamond kinematotropic chain are similar to that with RPRP kinematotropic chain. Equivalent revolute joint $R_{AC}$ of the diamond kinematotropic chain in branch (b) should be intersecting with the constraint force and connected to $R_2$ directly in F-limb configuration. Sixteen $L^F_C$ reconfigurable limbs constructed with diamond kinematotropic chain are listed in Table 4, where D denotes the diamond kinematotropic chain. Two configurations of the $R_1PR_1R_2-D$ reconfigurable limb are illustrated in Fig. 7 in which Fig. 7(a) shows a C-limb while Fig. 7(b) shows a F-limb.

![Table IV](image)

**TABLE IV.** $L^F_C$ reconfigurable limb structures with diamond kinematotropic chain.

IV. Reconfigurable Parallel Mechanisms

Taking three identical $L^F_C$ reconfigurable limbs from Table 2, Table 3 or Table 4 to connect a moving platform and a fixed base, a class of reconfigurable parallel mechanisms with various configurations can be obtained. A $L^F_C$ reconfigurable limb provides either a constraint force or a constraint couple to the moving platform with the kinematotropic chain in different motion branches, which determines that the mechanisms have the ability to
Fig. 9. Four configurations of the 3R1PR1R2-RPRP reconfigurable parallel mechanism. (a) 3T configuration; (b) 2T1R configuration; (c) 1T2R configuration; (d) 3R configuration

perform different kinds of motion. The 3R1PR1R2-FR, and 3R1PR1R2-RPRP parallel mechanisms are given as examples as shown in Fig.8 and Fig.9, respectively. In these mechanisms, when the \( L^c \) reconfigurable limbs evolve into F-limb configurations, the equivalent revolute joints of the kinematotropic chains and revolute joints \( R_2 \) intersect at a common point \( O \).

The limbs of the parallel mechanisms in Fig.8(a) and Fig.9(a) are all in C-limb configurations. Three limbs in each mechanism exert three constraint couples so that three rotational degrees of freedom of the moving platform are constrained. Each parallel mechanism has pure translational motion.

If one of the three reconfigurable limbs of a parallel mechanism evolves into F-limb configuration, there are two C-lims and one F-limb in each parallel mechanism as in Fig.8(b) and Fig.9(b). The limbs in each mechanism exert two constraint couples and one constraint force on the moving platform. Two rotational and one translational degrees of freedom of the moving platform are constrained. Each mechanism has two translational and one rotational motion.

If the parallel mechanisms reconfigure into the configurations with two F-lims and one C-limb in each mechanism as in Fig.8(c) and Fig.9(c), two constraint forces passing through point \( O \) and one constraint couple are exerted on the moving platform. Each mechanism has two rotational and one translational motion in these configurations.

When all the three limbs in each mechanism evolve into F-limb configurations as in Fig.8(d) and Fig.9(d), three constraint forces passing through point \( O \) are exerted on the moving platform in each mechanism. Three translational degrees of freedom of the moving platform are constrained and the mechanisms become spherical parallel mechanisms in these configurations.

So for these parallel mechanisms, with the kinematotropic chains in the \( L^c \) reconfigurable limbs evolving into different motion branches, the moving platform has the ability to perform different kinds of motion including 3T motion, 2T1R motion, 2R1T motion and 3R motion.

As aforementioned, in order to actuate the kinematotropic chain switching between different motion branches, two actuators should be mounted to each kinematotropic chain. There will be six actuators in each mechanism of this class. If the actuate joints are rigidized, the three reconfigurable limbs in each mechanism will degenerate into three \( L^c \) limbs, which exert three constraint forces and three constraint couples on the moving platform. These constraints form a 6-system and the mechanism will loss all the mobility, which satisfy the
actuation validity conditions for parallel mechanisms [23]. In other words, the six actuators can sufficiently drive the configuration switch of the parallel mechanism, leading to degrees of freedom variation of the platform. All the actuators are nearly mounted to the base, which will benefit the dynamic performance of this kind of mechanisms.

V. Conclusions

In this paper, three kinematotropic chains characterized by change in pair connectivities without change of the number of degrees of freedom are introduced into the design of reconfigurable kinematic limbs. Taking a L_{FC}
limb together with one of the three kinematotropic chains, reconfigurable limb structures are obtained with some necessary requirements satisfied. With the kinematotropic chain in different motion branches, a reconfigurable limb has two configurations in which a constraint couple and a constraint force are exerted, respectively. Using three identical reconfigurable limbs to connect a moving platform and a fixed base, a class of reconfigurable parallel mechanisms is obtained. With the reconfigurable limbs evolving into different configurations, these mechanisms have the ability to perform various motion modes such as 3T motion, 2T1R motion, 2R1T motion and 3R motion. Finally, the actuation scheme for these mechanisms is discussed. It shows that six actuators mounted to the three kinematotropic chains in each mechanism can actuate the mechanism switching between different configurations and performing different kinds of motion. The proposed reconfigurable parallel mechanisms have the potential to be applied in production with various task requirements.

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References