Modeling of Cubic-Stewart vibration isolation platform with hybrid vibration isolator

Y. Zhang¹
Harbin Institute of Technology
Harbin, China

Z.-B. Chen²
Harbin Institute of Technology
Harbin, China

Y.-H. Jiao³
Harbin Institute of Technology
Harbin, China

Abstract—In order to protect the instrument and equipment for space applications, vibration control is necessary. In this paper, a Cubic-Stewart vibration isolation platform employing a novel hybrid vibration isolator (HVI) is presented. The HVI is composed of the active piezoelectric stack actuator and the passive rubber isolator, which features compact structure and high reliability. The simulation results the active piezoelectric stack actuator can eliminate the resonance peak significantly and the passive rubber isolator is effective to isolate a part of vibration once active control fails. Finally, the simulation analysis of the Cubic-Stewart platform is established based on the transmissibility and single input single out (SISO) control algorithm. The results verify the effectiveness of our proposed vibration isolation system.

Keywords: vibration isolation, piezoelectric-stack actuator, hybrid vibration isolator, Cubic-Stewart platform

I. Introduction

Vibration isolation and control have become a widespread and significant challenge in spacecraft. The various vibration source of the spacecraft impacts the sensitive instrument seriously, such as vibration generated by flywheel attitude adjustment, solar array drives, refrigeration equipment and shock loads and nonlinear vibrations generated by completing a particular action[1]. At present, most of the sensitive instruments which required vibration isolation mainly used passive damping isolation or flexible structure. The passive vibration isolation cannot meet the higher demand because of its small vibration isolation frequency range, small vibration attenuation, and high frequency vibration suppression with low frequency vibration amplification [2, 3]. As the stricter requirements in the aerospace Engineering, the passive vibration isolation systems alone cannot often be satisfied. In contrast, the active vibration isolation can overcome these insufficiencies, but it has high requirement to the control system design. Once any the control problems will lead to the failure of the whole isolation system [4], so the system stability of the active vibration isolation is poor.

With the development of the material technology and advance control technology, the hybrid vibration isolation systems have attracted a lot of attention which employ the passive isolation and the active control isolation together, because they can combine with the advantages of active vibration isolation and passive vibration isolation. Nakamura et al. developed a hybrid actuator, which was composed of an air actuator and a giant magnetostrictive actuator in series configuration [5, 6]. Nguyen proposed a hybrid active mount featuring piezoelectric stack and a rubber element that can provide high performance in terms of vibration control [7]. Jang et al. developed a hybrid mount system with air springs and piezoelectric stack actuators for microvibration control [8].

As a popular parallel mechanism invented by Gough and summarized by Stewart[9], Stewart platforms have wide-range application such as flight simulators, vehicle simulators, vibration tables, and space dock mechanisms [10]. One of such systems called “Cubic Configuration” was invented by Geng and used by Intelligent Automation Inc.[11]. The cubic configuration has several unique characteristics, making such Stewart platform capable of decoupled control, since adjacent legs are orthogonal to each other, as well as using the symmetry of cube to provide identical legs [12]. The features of the Cubic-Stewart would be useful in vibration isolation control.

In this paper, aiming to utilize the advantages of the passive vibration and the active control, a hybrid vibration isolator is developed. In our design, the vulcanized rubber and the piezoelectric stack are combined in the series configuration. In that manner, the advantages of the passive and active vibration systems can be utilized simultaneously. In addition, our proposed HVI has simple and compact structure, small quality and size, small energy consumption, which are distinct features and advantages when the HVI is used in the aerospace engineering. Because of its compactness, the HVI is suitable for application on multi-dimensional platform, the HVI is applied on the Cubic-Stewart vibration isolation platform in this paper. The joints which connect platform and the HVI adopt omnidirectional type flexible hinge to avoid the clearance/ friction/ crawling problems of the traditional hinge, improve the control precision, realize multi degree of freedom vibration control of sensitive instruments.

¹zy@hit.edu.cn
²chenzb@hit.edu.cn
³jiaoyh@hit.edu.cn
II. Hybrid vibration isolator

A. Mechanism design of the hybrid isolator

To achieve the active and passive vibration isolation together, our proposed hybrid isolator shown in Fig. 1 consists of an active part and a passive part, which are combined in serial connection. As the active part of the hybrid system, a piezoelectric stack actuator is selected. Piezoelectric stack actuators have several suitable properties for microvibration control, such as a fast response time, high dynamic force generation, and easiness of control due to the linearity between the input and output.

Of the various passive devices, the rubber is used in this study because of its several features, such as sample structure, high damper, easy to metal bonding.

To describe the mechanical design of the HVI clearly, the HVI structure is divided into two subsystems: the active and passive vibration isolation structures. The two parts are connected in series configuration in order to make full use of their isolation performances, i.e. the leveling function of the vulcanized rubber and the fast response characteristics of the piezoelectric stack actuator. The active vibration isolation structure contains the piezoelectric stack actuator, the adjustment parts, force output rod, and the pre-loaded spring. The linear bearing outside of the force output rod is used to guarantee the axial displacement of the piezoelectric stack. With the pre-loaded spring, the piezoelectric stack is always compressed to avoid the radial displacement and the structure gap. The passive structure is composed of the vulcanized rubber and the force transmission rod. The specifications of the HVI are summarized in Table I.

<table>
<thead>
<tr>
<th>Element</th>
<th>Item</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber</td>
<td>Size (H, H0, H1, R1, R2)</td>
<td>29mm, 28mm,</td>
</tr>
<tr>
<td></td>
<td>H0 (Hardness JIS)</td>
<td>40 H</td>
</tr>
<tr>
<td></td>
<td>Gs (Static shear modulus)</td>
<td>0.45 N/mm²</td>
</tr>
<tr>
<td>Piezoelectric stack actuator</td>
<td>Maximum stroke length</td>
<td>36um</td>
</tr>
<tr>
<td></td>
<td>Maximum force output</td>
<td>1400N</td>
</tr>
<tr>
<td></td>
<td>Maximum tension</td>
<td>300N</td>
</tr>
<tr>
<td>Hybrid</td>
<td>Input voltage</td>
<td>-30~+150v</td>
</tr>
<tr>
<td></td>
<td>Krev</td>
<td>20mm</td>
</tr>
</tbody>
</table>

TABLE I. Specifications of the HVI

B. Dynamic model of the hybrid isolator

The displacement of the piezoelectric stack actuator can be formulated by,

$$x = \frac{F_L}{EA} + d_{13}U$$

(1)

where the $F_L$, $E$, $A$, $L$, and $d_{13}$ are the driving force, the modulus of elasticity, the action area, the length, the driving voltage and the piezoelectric stress constant of the piezoelectric stack, respectively.

Based on the above analysis, we can build the dynamic model of the HVI with the mass-spring model illustrated in Fig. 2. According to the Newton’s second law, the system equilibrium equation can be formulated by,

$$m_1\ddot{x}_1 + k_1(x_1 - x_0) = -F_i$$

$$m_2\ddot{x}_2 + c(x_2 - x_0) + k_2(x_2 - x_0) + k_1(x_1 - x_0) = F_i$$

(2)

where $m_1$ is the mass of load and $m_2$ is middle mass, namely, the mass of the piezoelectric stack and the adjustment, etc. $x_1$ and $x_2$ are the displacements of the external load and the middle mass. $k_1$ and $k_2$ are the stiffness of the piezoelectric stack actuator and the rubber. $x_0$ is the displacement of the disturbance. The $c$ is the damping coefficient of the rubber. $F_i$ is the output force of the piezoelectric stack actuator.

The Eq. (2) can be rewritten in the form of the equation of state.

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{k_1}{m_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & \frac{k_3}{m_2} & \frac{c_1}{m_2} & \frac{1}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{m_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} F_i + \begin{bmatrix} d_{13} \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

(3)

Based on the dynamic model, the system can be formulated by the state equation and output equation, where the acceleration of the load platform is measured as the system output, the state equation (3) and the output equation can be written to simplify the notation by,

$$\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} = A\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + B\begin{bmatrix} u(t) \\ w(t) \end{bmatrix} + G\begin{bmatrix} \xi(t) \end{bmatrix}$$

$$y(t) = C\begin{bmatrix} x(t) \end{bmatrix} + Du(t)$$

(4)

where $x(t)$ is the time-varying state variables $x = [x_1 \ x_2 \ \dot{x}_1 \ \dot{x}_2]^T$. $u(t)$, $w(t)$ and $y(t)$ are the control variables, the disturbance and the output. $A$, $B$, $G$ are the
parameter matrices of the Eq. (3). \( C \) and \( D \) can be expressed as
\[
C = \begin{bmatrix}
-k_{c1} & k_{c2} & 0 & 0
\end{bmatrix}, \quad D = \begin{bmatrix}
-\frac{1}{m}
\end{bmatrix}.
\]

C. Controller of the hybrid isolator

A suitable control strategy plays a very important role in the active vibration control. The linear quadratic regulator (LQR) is employed to simulate the effect of our isolator in this paper, because the LQR control method can provide inherent robustness and simplicity in design and implementation. The vibration isolation objective is to decrease the acceleration response of the load platform. With the consideration of the vibration isolation performance and the stability of the system, the objective function is formulated as follows,

\[
J = \int_{0}^{\infty} \left( X^T Q X + U^T R U \right) dt
\]

where \( Q \) and \( R \) are the state weight matrix and the control weight matrix. We determine \( Q \) and \( R \) according to the empirical value. The state feedback controller gain matrix \( L \) can be solved by LQR method, and

\[
U = -LX
\]

where the \( L \) is the feedback gain matrix.

The \( P \) can be solved form the Riccati algebra equation of the controller,

\[
PA + A^T P - PBR^{-1}B^T P + Q = 0
\]

Let \( L = [k_{c1}, k_{c2}, k_{c3}, k_{c4}] \) represent the feedback gain of the LQR controller. The active control force of the piezoelectric stack actuator can be formulated by,

\[
F_c = k_{c1} x_1 + k_{c2} x_2 + k_{c3} x_3 + k_{c4} x_4
\]

Substitute Eq.(9) into Eq.(4) and use Laplace transform to formulate the system transfer functions of the passive control (open-loop, \( H_1 \)) and the integrated passive and active vibration control (IPAVC) (closed-loop, \( H_2 \)).

\[
H_1 = \frac{x_2(s)}{u(s)} = \frac{k_{c1} s + k_{c2}}{m_2 s^2 + a_1 s + k_{c1} s + k_{c2}}
\]

\[
H_2 = \frac{x_2(s)}{u(s)} = \frac{c k_{x3} s^2 + a_2 s + (k_{c1} k_{x3} + k_{c2} k_{x4})}{m_2 s^2 + a_1 s + a_3 s + (k_{c1} k_{x3} + k_{c2} k_{x4})}
\]

where

\[
a_1 = m_2 k_{x3} + m_2 k_{x4} ; \quad a_2 = c k_{x3} s + c k_{x4} s ;
\]

\[
a_3 = m_2 k_{x3} + m_2 k_{x4} + m_2 k_{x1} + m_2 k_{x2} + c k_{x1} s + c k_{x2} s ;
\]

\[
a_4 = -k_{c1} m + c m_1 + m_2 k_{x2}.
\]

D. Simulation for the hybrid isolator

In order to test the vibration isolation capability of the HVI system, a simulation model based on LQR control is established in MATLAB/SIMULINK. Two sinusoidal vibration signals with the frequency of 20Hz, 20 um and 200Hz, 20 um are acted on the base platform, respectively. The open-loop responses and the closed-loop responses of the load platform are shown in Fig.3. The disturbances can be eliminated by about 80% in 20Hz, and almost 100% in 200 Hz. The vibration transmission characteristic curves are shown in Fig. 4. The IPAVC can basically eliminate the fist-order vibration peak and passive control plays the main role at the high frequency.

![Fig. 3](image)

Fig. 3. The open-loop and closed-loop responses of the load platform with different disturbances: a) The disturbance of 20um, 20Hz sine signal. b) The disturbance of 20um, 200Hz sine signal disturbance.

![Fig. 4](image)

Fig. 4. The open-loop and closed-loop vibration transmission characteristic.

III. Cubic-Stewart vibration isolation system

A. Transfer characteristics of the vibration isolation system

Generalized coordinates of the vibration isolation system based on Cubic-Stewart platform are shown in Fig. 5. Our proposed hybrid vibration isolator is employed as the support leg of Cubic-Stewart platform, which is arranged in a mutually orthogonal configuration.
connecting the corners of a cube. Any two legs are orthogonal or parallel, each direction of the leg is a set of generalized coordinates. The topology provides a uniform control capability and a uniform stiffness in all directions, and it minimizes the cross-coupling amongst actuators and sensors of different support legs (being orthogonal to each other) [13]. Based on those features, the SISO control method can be adopted for our proposed Cubic-Stewart vibration isolation system.

As depicted in Fig. 5, the node R, S, T and U, V, W is the platform joint respectively. The base frame \( \{x, y, z\} \) has its origin at node O and the payload frame \( \{x', y', z'\} \) has its origin at node O'. The legs US and WT are parallel with x axis. WR and VS are parallel with y axis. UR and VT are parallel with z axis.

It is assumed that each actuator is actuated along the corresponding axis direction when the vibrations are acted on the base platform. Let \( x'_U, x'_W, y'_R, y'_V, z'_U, z'_V \) and \( x_U, x_W, y_R, y_V, z_U, z_V \) express the generalized coordinates of the joints in the base platform and the payload platform, respectively. In the \( O-x'y'z' \) coordinates, the position and the pose of the base platform can be expressed by generalized coordinates of the joints in the base platform,

\[
\begin{align*}
x &= (x_U + x_W) / 2 \\
y &= (y_R + y_V) / 2 \\
z &= (z_U + z_V) / 2 \\
\theta_x &= (z_U - z_V) / l + (y_V - y_U) / l \\
\theta_y &= (x_U - x_W) / l + (z_U - z_V) / l \\
\theta_z &= (y_U - y_V) / l + (x_W - x_U) / l
\end{align*}
\]

(11)

where \( l \) is the length of each leg, and Eq. (11) can be re-written in matrix form,

\[
\begin{bmatrix}
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 \\
-1 & 0 & 0 & -1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 \\
-1 & 0 & -1 & 0 & 1 & 0 \\
\end{bmatrix} x = M \begin{bmatrix}
x_U \\
y_U \\
z_U \\
x_W \\
y_W \\
z_W \\
\end{bmatrix} = M \begin{bmatrix}
x'_U \\
y'_U \\
z'_U \\
x'_W \\
y'_W \\
z'_W \\
\end{bmatrix}
\]

(12)

where \( M \) is defined as the coordinate transformation matrix.

In the same manner, the position and pose of the payload platform which are described in the \( O-x'y'z' \) coordinates, can be expressed by the generalized coordinates of the joints in the payload platform.

\[
\begin{align*}
x' &= (x'_U + x'_W) / 2 \\
y' &= (y'_R + y'_V) / 2 \\
z' &= (z'_U + z'_V) / 2 \\
\theta' &= (z'_U - z'_V) / l + (y'_V - y'_U) / l \\
\theta'_x &= (x'_U - x'_W) / l + (z'_U - z'_V) / l \\
\theta'_z &= (y'_U - y'_V) / l + (x'_W - x'_U) / l
\end{align*}
\]

(13)

Eq. (13) can be re-written in matrix form.

\[
\begin{bmatrix}
\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 \\
-1 & 0 & 0 & -1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix} x' = M \begin{bmatrix}
x'_U \\
y'_U \\
z'_U \\
x'_W \\
y'_W \\
z'_W \\
\end{bmatrix} = M \begin{bmatrix}
x'_S \\
y'_S \\
z'_S \\
x'_T \\
y'_T \\
z'_T \\
\end{bmatrix}
\]

(14)

For each leg, based on the transfer characteristics \( \{H(s)\} \) among the input and the output of the actuator established in Eq. (11), the transfer characteristics of the Cubic-Stewart vibration isolation system can be derived as shown in Fig. 6.

\[
\begin{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\end{bmatrix}
\]

Fig. 6 The transfer characteristics of the vibration isolation system

B. Numerical simulation

According to the transfer characteristics, the simulation analysis of the Cubic-Stewart platform is established in the MATLAB/SIMULINK, in which the vibration isolation performance under different excitation signals are analyzed. Two sets of excitation signals are acted on the base platform, i.e., the sinusoidal excitation signal and the step excitation signal.

Firstly, the vertical and horizontal sinusoidal vibration signals with the frequency of 10Hz, 10am are acted on the base platform respectively. The displacements of the payload platform are obtained to validate the effectiveness of our proposed vibration isolation system. The results show that our proposed system presents excellent vibration
isolation effect for horizontal and vertical direction low frequency harmonic excitation signals, as shown in Fig. 7 and Fig. 8. Compared to the passive control, the vibration amplitude of the active control is attenuated by 80% in vertical direction and 50% in horizontal direction. The attenuations of the angular displacements are much more obvious.

When the vertical and horizontal step signals with the amplitude of 10\( \mu \)m are acted on the base platform, the corresponding displacements of the payload platform are shown in Fig. 8 and Fig. 9 respectively. The results show that the platform proposed can reduce the vibration displacement and angular displacement and greatly reduce the number of the oscillation. Therefore the steady state of the system can be realized quickly.

---

**Fig. 7** The vibration isolation effect for the vertical harmonic signal excitation: (a) Linear displacement. (b) Angular displacement.

**Fig. 8** The vibration isolation effect for the horizontal harmonic signal excitation: (a) Linear displacement. (b) Angular displacement.

**Fig. 9** The vibration isolation effect for the vertical step signal excitation: (a) Linear displacement. (b) Angular displacement.

**Fig. 10** The vibration isolation effect for horizontal step signal excitation: (a) Linear displacement. (b) Angular displacement.
IV. Conclusions
In this paper, we developed a Cubic-Stewart vibration isolation platform employing a hybrid vibration isolator. In our design, the hybrid vibration isolator is composed of the rubber isolator and the piezoelectric stack actuator. LQR control strategy is developed for the active vibration isolation of the HVI. The simulation results have shown that HVI can isolate 80% vibration. The piezoelectric stack actuator can eliminate the resonance peak significantly and the rubber isolator can isolate the high frequency vibration and ensure the reliability of the HVI. Using the feature of the Cubic-Stewart that each coordinate direction of motion is linearly independent and the coupling factors greatly reduce, the transmissibility and distributed control model is built up, then the vibration isolation performance is simulated in MATLAB/SIMULINK. The simulation results show that the vibrations on the load platform can be significantly reduced by the Cubic-Stewart platform employing the HVI.

However, our proposed works in this paper are preliminary, much more works will be carried out. In follow-up study, the experiments will be conducted to verify the dynamic model and vibration isolation performance of the HVI and Cubic-Stewart vibration isolation system.

Acknowledgments
This research is funded by the National Natural Science Foundation of China (Grant No.11172075).

References