Trajectory Control of Curve Constrained Hyper-Redundant Space Manipulator

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Abstract: This paper presents a methodology to control motion planning of a hyper-redundant planar pace robot tracing a specified end-effector path. For this, a space robot of 6 DOF has been considered for study. A hyper-redundancy enhances the capability to realize various kinds of task, i.e., obstacle avoidance, joint limits, singularity avoidance, etc. For full utilization of its potential capability, shape control is suggested here; that is, the tip of the links of manipulator thus its entire body is controlled by fitting end of each link onto a provided time varying curve. For shape control, the control objectives are formulated defining a contour correspondence between a hyper-redundant robot and a curve that lay down a desired shape. The whole manipulator body is to be stayed on onto a spatial curve. Some of the joints are locked using Proportional-Derivative (PD) control and the rest are kept active. Proportional-Derivative-Integral control has been used to take care of end-effector trajectory tracking in the workspace. The bond graph technique has been used for the dynamic model of the system and generating system equations.

Keywords: Space robot, Hyper-redundancy, Curve constraint, Trajectory control, Bond graph model.

1. Introduction

Hyper-redundant space manipulators consist of a very large degree of freedom (DOF) to execute a desired task. A greater number of DOF (n) than the required task space coordinates (m) makes redundant to the robot which is characterized with superior dexterity. Various tasks (unconventional features) such as manipulation in the narrow space, avoiding obstacles, singularity avoidance, joint limit avoidance, minimization of power consumption, task compatibility, etc., cannot be performed satisfactorily by a conventional space robot. These shortcomings can be overcome by hyper-redundant space robots. However, controlling of the hyper-redundant space robots is a tedious but challenging task which is increasingly attracting many researchers worldwide.

Whitne [1], originated first the concept of the pseudo-inverse of the Jacobian for redundancy resolution in the manipulator. Liegeois [2], included the concept of self-motion of the manipulator by the null space of the Jacobian matrix to extend the pseudo-inverse approach. Nenchev et al. [3] and Caccavale and Bruno [4], studied the pseudo-inverse redundancy resolution technique for the kinematic control of redundant space manipulators. They controlled the trajectory tracking as well as the Klein and Huang [5], examined problems related to pseudo-inverse control by showing that in some cases this control strategy leads singularity problem by giving undesired arm configurations. Nakamura and Mukherjee [6], discussed trajectory control scheme using the nonholonomic redundancy of 6-DOF free-flying robot to avoid obstacles and joint limits. Vafa and Dubowsky [7], firstly suggested the virtual manipulator to analyze the kinematics and dynamics of space manipulator. Sutar et al. [8], presented a concept of virtual link based controller for hyper-redundancy resolution to control the tip trajectory of the in-vivo robot used for surgical applications. Jareanpon et al. [9], proposed a shape control method of a hyper-redundant arm used to dock an object. This is done by using whole arm manipulation through encircling the object based on the virtual constraints on each link. Mochiyama et al. [10], discussed dynamics based shape control of a hyper-redundant manipulator. Shape control provides fundamental control for whole arm manipulation [11]. The aforementioned literature motivated us to utilize unconventional features of the hyper-redundant space robot exploring a more general method.

In this paper, we consider to study trajectory control of curve constrained hyper-redundant space manipulator. For this, we propose two basic shape control problems for the dynamic model of hyper-redundant space robot. One is shape regulation and the other is shape tracking. The former tries to bring the manipulator into a desired shape. The latter attempts to make the manipulator followed a moving desired shape [12]. In both cases, we need an apparatus to prescribe a desired shape. For this, a circular curve as a constraint for whole arm manipulation has been proposed. The link end is brought onto a curve by locking some of the joints through the PD controller and actively controlling the other joints. For this, the physical model of 6 DOF manipulator has been developed for its kinematic and dynamic analysis. The proposed control has been implemented in different cases such as in joint space as well as workspace using a limited number of joint actuation depending upon requirements. The circular and the Bezier curve are used as reference tip trajectory to validate the idea.

This paper is organized as follows. Section II deals with a model of the 6 DOF planar hyper-redundant space manipulator using bond graph. It includes a model of a free-floating body. Section III discusses the development of control strategies and its implementation in joint space and workspace considering different cases. It also includes evaluation of the Jacobian and the PID controller. Section IV presents simulation and animation results. Finally, section V deals with concluding remarks with future scope.

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II. Model of 6DOF Planar Hyper-Redundant Space Manipulator

Modeling of 6DOF planar hyper-redundant space manipulator involves linear as well as rotational dynamics of the links. The modeling is based on assumptions that all the links and space robot base are composed of rigid bodies. Also, the robot has a single arm with revolute joint, and it is an open kinematic chain. Figure 1 shows a schematic representation of the hyper-redundant planar space robot. In this Figure, \( \{A\} \) is the absolute frame and \( \{V\} \) is the vehicle frame located at the center of mass (CM) of space robot. A frame \( \{0\} \) is located on space robot base at the root of the manipulator of space robot. Frame \( \{1\} \) is located at the root of manipulator at the first joint. Frames \( \{2\}, \{3\}, \{4\}, \{5\} \) and \( \{6\} \) are located at 2nd, 3rd, 4th, 5th and 6th joint angles of manipulator, respectively. Frame \( \{7\} \) is located at the tip of the manipulator. Let, \( r \) is the distance between the vehicle frame and the root of the frame for the space robot. Let, \( l_1, l_2, l_3, l_4, l_5, l_6 \) and \( l_7 \) are length of 1st, 2nd, 3rd, 4th, 5th and 6th links of manipulator, respectively.

![Figure 1. Schematic representation of hyper-redundant planar space robot](image)

Let \( \phi \) represents the rotation of the frame \( \{V\} \) with respect to the frame \( \{A\} \). Let \( \theta_1, \theta_2, \theta_3, \theta_4, \theta_5 \) and \( \theta_6 \) show 1st, 2nd, 3rd, 4th, 5th and 6th joint angles of manipulator, respectively. Let \( X_{CM} \) and \( Y_{CM} \) are the space robot’s CM coordinates with respect to the absolute frame \( \{A\} \). Kinematic relations are expressed in terms of the tip position and orientation as:

\[
X_{tp} = X_{CM} + r \phi + l_1 c(\theta_1) + l_2 c(\theta_12) + l_3 c(\theta_{123}) + l_4 c(\theta_{1234}) + l_5 c(\theta_{12345}) + l_6 c(\theta_{123456}) \tag{1}
\]

\[
Y_{tp} = Y_{CM} + r s(\phi) + l_1 s(\theta_1) + l_2 s(\theta_{12}) + l_3 s(\theta_{123}) + l_4 s(\theta_{1234}) + l_5 s(\theta_{12345}) + l_6 s(\theta_{123456}) \tag{2}
\]

Where, \( \theta_{12} = \phi + \theta_1, \theta_{123} = \theta_1 + \theta_2, \theta_{1234} = \theta_1 + \theta_2 + \theta_3, \theta_{12345} = \theta_1 + \theta_2 + \theta_3 + \theta_4, \theta_{123456} = \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5, \)

and, \( \theta_{123456} = \theta_{12345} + \theta_6 \)

\[
\theta_{op} = \theta_{123456} = \phi + \theta_1 + \theta_2 + \theta_3 + \theta_4 + \theta_5 + \theta_6 \tag{3}
\]

Here, \( s(\phi) \) and \( c(\phi) \) represent \( \sin(\phi) \) and \( \cos(\phi) \), respectively. The tip translational and the angular velocities can be evaluated with help of equations (1), (2) and (3) as,

\[
\dot{X}_{tp} = \dot{X}_{CM} - r \dot{\phi} s(\phi) - l_1 \dot{\theta}_1 c(\theta_1) - l_2 \dot{\theta}_{12} c(\theta_{12}) - l_3 \dot{\theta}_{123} c(\theta_{123}) - l_4 \dot{\theta}_{1234} c(\theta_{1234}) - l_5 \dot{\theta}_{12345} c(\theta_{12345}) - l_6 \dot{\theta}_{123456} c(\theta_{123456}) \tag{4}
\]

\[
\dot{Y}_{tp} = \dot{X}_{CM} + r \dot{\phi} c(\phi) + l_1 \dot{\theta}_1 c(\theta_1) + l_2 \dot{\theta}_{12} c(\theta_{12}) + l_3 \dot{\theta}_{123} c(\theta_{123}) + l_4 \dot{\theta}_{1234} c(\theta_{1234}) + l_5 \dot{\theta}_{12345} c(\theta_{12345}) + l_6 \dot{\theta}_{123456} c(\theta_{123456}) \tag{5}
\]

\[
\dot{\theta}_{op} = \dot{\theta}_{123456} = \dot{\phi} + \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4 + \dot{\theta}_5 + \dot{\theta}_6 \tag{6}
\]

Equations (4), (5) and (6) help in evaluating transformer moduli for drawing bond graph model of robot systems as shown in Figure 2. These transformer moduli are shown in Table 1. In Figure 2, I elements are used to model translational and rotational inertia of the space robot system. The \( R \) element represents the damping present. In Figure 2, \( X_t \) and \( Y_t \) denote the body-fixed coordinates and the modeling of space robot is done in a non-inertial frame. The base of robot rotates about Z-axis. For a planar case, where, \( \alpha_0 = 0, \alpha_3 = 0 \) and \( v_z = 0 \), the net force acting on manipulator can be given as [13],

\[
F_x = M \ddot{x}_{CM} - M_v \dot{\phi} \dot{y}_{CM}, \tag{7}
\]

\[
F_y = M_v \ddot{y}_{CM} + M \dot{\phi} \dot{x}_{CM}, \tag{8}
\]

\[
F_z = 0. \tag{9}
\]

III. Development of Control Strategy

In our proposed control strategy, a circular curve is used to exploit redundancy based on the concept of whole arm manipulation. However, it is not restricted to circular curve only, it can also be valid for others. As per this, the backbone of hyper-redundant manipulator falls onto an arc of a circle.

The general form of circle equation can be written as,

\[
(x - x_c)^2 + (y - y_c)^2 = a^2 \tag{10}
\]

Where, \( x_c \) and \( y_c \) are centre coordinates and \( a \) is radius of circle. Here, our concept is to use the link ends of the hyper-redundant space robot to fit onto a circular arc and then perform the motion planning of the arc. For this, three control points are needed for circular curve calculation. These points are defined from a three coordinates of 6 DOF planar robot. Let us assume that \( r, l_1, l_2, \phi, \theta_1 \), and \( \theta_2 \) are known. Let, \( r = l_1 = l_2 = 0.5, \phi = 0 , \theta_1 = 45^\circ \) and \( \theta_2 = 30^\circ \). From this assumed values, coordinates of three required points to get the value of \( x_c, y_c \) and \( a \) can be evaluated as,
For simplicity, one can find remaining joint coordinates as, 

\[
(x_2 - x_1)^2 + (y_2 - y_1)^2 = a^2  
\]  

(15)

\[
(x_3 - x_2)^2 + (y_3 - y_2)^2 = a^2  
\]  

(16)

With the help of equations 14, 15 and 16, centre coordinates \((x_i, y_i)\) and radius \(a\) can be expressed as a function of \((x_1, y_1, x_2, y_2, x_3, y_3)\), these are:

\[
x_c = \frac{a_1}{2a_2}, y_c = \frac{h_1}{2b_2}  
\]  

(17)

\[
a = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}  
\]  

(18)

\[
a_1 = -(x_3^2 + y_3^2 - x_2^2 - y_2^2 - 2y_3^2)y_1 + (x_3^2 + y_3^2)  
\]

\[-x_2^2 - y_2^2)x_1 - (x_3^2 + y_3^2)y_3  
\]

(19)

\[
a_2 = (y_3 - y_2)x_1 - (x_3 - x_2)y_1 + x_3y_2 - x_2y_3  
\]

\[b_1 = -(x_3^2 + y_3^2 - x_2^2 - y_2^2)y_1 + (x_3^2 + y_3^2)  
\]

\[-x_2^2 - y_2^2)x_1 - (x_3^2 + y_3^2)y_3  
\]

(19)

\[
b_2 = (2x_1 - x_2 - x_3)y_1 + (x_3 - x_2)y_2 + (x_2 - x_1)y_3  
\]

With the help of coordinates of the center and radius of the circle, for simplicity, one can find remaining joint angles, i.e., \(\theta_1, \theta_2, \theta_3\) and \(\theta_4\) from Figure and its corresponding tip coordinates of links 3, 4, 5 and 6. The tip coordinates can also be evaluated analytically by
assuming all links of equal length and substituting \( \theta_i = \theta_k = \theta_l \) in equations (19)-(23),

\[
x_i = x_i + l_c(\theta_{\text{g23}}), \quad y_i = y_i + l_s(\theta_{\text{g23}})
\]

\[
x_4 = x_4 + l_c(\theta_{\text{g234}}), \quad y_4 = y_4 + l_s(\theta_{\text{g234}})
\]

\[
x_5 = x_5 + l_c(\theta_{\text{g2345}}), \quad y_5 = y_5 + l_s(\theta_{\text{g2345}})
\]

\[
x = X_{\text{tip}} = x_5 + l_c(\theta_{\text{g23456}}), \quad y = Y_{\text{tip}} = y_5 + l_s(\theta_{\text{g23456}})
\]

**In conventional robot, all joints must be actuated to reach around the obstacle. The hyper-redundant manipulator can achieve the same motion by actuating a limited number of joints. Expanding this concept to 6 DOF planar space robot, one can see the advantage of hyper-redundant manipulator. In order to test the efficacy of the proposed control strategy for constrained hyper-redundant manipulator, two examples are developed in followings:**

**A. Example 1: Joint Trajectory of 6 DOF Planar Space Robot in Joint Space**

This example deals with two cases, i.e., the joint trajectory of 6 DOF space robot in joint space with one actuated joint (Case A) and two actuated joints (Case B). In the former case, joint 1 \((\theta_1)\) is actuated while in later case joint 1 and joint 4 \((\theta_1 \text{ and } \theta_4)\) are actuated. Exploiting redundancy, one can actuate any joint as per specific requirement. To control joint motion, PD (Proportional Derivative) control is used as shown in Fig. 2. The control strategy for the actuator at the joint can be given as,

\[
\tau = K_p (\theta_0 - \theta_a) - K_d \dot{\theta}_a
\]

Where, \( \tau \) is the joint torque, \( \theta_0 \) is the desired position of joint and \( \theta_a \) is the actual position of joint, \( \dot{\theta}_a \) is the actual joint angular velocity of joint and \( K_p \) and \( K_d \) are the proportional and the derivative gain parameters. One can enforce a joint to be laid onto a circular arc by giving the desired angle as determined from Fig. 3.

**B Example 2: Tip Trajectory of 6 DOF Planar Space Robot in Workspace**

This example also deals with two cases, i.e., tip trajectory control of 6 DOF space robot in workspace with all actuated joints (Case C) and two actuated joints (Case D). In the latter case, joint 1 and joint 4 \((\theta_1 \text{ and } \theta_4)\) are actuated to achieve the desired motion as shown in Figure 2. In this figure, evaluation of Jacobian has been represented in the signal form. This bond graph model can be extended for all actuated joints by incorporating Jacobian to the remaining joints i.e., \( \theta_2, \theta_3, \theta_5 \) and \( \theta_6 \). For mapping joint motion into tip motion, the Jacobian is used and it is described below in both the cases.

**B1. Evaluation of Jacobian for all actuated joints**

The Jacobian converts the effort signals to joint torques. The gains required to model Jacobian from kinematics relations as,

\[
\begin{bmatrix}
\dot{X}_{\text{tip}} \\
Y_{\text{tip}}
\end{bmatrix} =
\begin{bmatrix}
\dot{X}_{\text{cm}} - r \phi \phi \phi \\
Y_{\text{cm}} + r \phi \phi \phi
\end{bmatrix}
\]

\[
+ [\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10} ]^T \theta
\]

\[
= \begin{bmatrix}
\dot{\alpha_1} & \dot{\alpha_2} & \dot{\alpha_3} & \dot{\alpha_4} & \dot{\alpha_5} & \dot{\alpha_6} & \dot{\beta_1} & \dot{\beta_2} & \dot{\beta_3} & \dot{\beta_4} & \dot{\beta_5} & \dot{\beta_6} & \dot{\beta_7} & \dot{\beta_8} & \dot{\beta_9} & \dot{\beta_{10}}
\end{bmatrix}
\]

Where,

\[
\alpha_i = \alpha_i - l_s(\theta_{\text{g23456}}), \quad \alpha_6 = \alpha_5 - l_s(\theta_{\text{g23453}}),
\]

\[
\alpha_1 = \alpha_1 - l_s(\theta_{\text{g234}}), \quad \alpha_6 = \alpha_5 - l_s(\theta_{\text{g2345}}),
\]

\[
\alpha_1 = \alpha_1 - l_s(\theta_{\text{g234}}), \quad \alpha_5 = \alpha_5 - l_s(\theta_{\text{g2345}}),
\]

\[
\alpha_6 = -l_s(\theta_{\text{g234}}), \quad \beta_1 = \beta_1 + l_c(\theta_{\text{g2345}}),
\]

\[
\beta_1 = \beta_1 + l_c(\theta_{\text{g234}}), \quad \beta_5 = \beta_5 + l_c(\theta_{\text{g2345}}),
\]

\[
\beta_6 = l_c(\theta_{\text{g234}}), \quad \beta_7 = l_c(\theta_{\text{g2345}}),
\]

**B2. Evaluation of Jacobian for a limited number of actuated joints**

In this case where joint 1 and joint 4 are actuated, the kinematic relations for the tip displacement \( X_{\text{tip}} \) and \( Y_{\text{tip}} \) in \( X \) and \( Y \) directions can be derived from Figure 3. These are,

\[
X_{\text{tip}} = r c(\phi) + l_{46} c(\theta_{\text{g1}}) + l_{46} c(\theta_{\text{g114}})
\]

\[
Y_{\text{tip}} = r s(\phi) + l_{46} s(\theta_{\text{g1}}) + l_{46} s(\theta_{\text{g114}})
\]

\[
\theta_{\text{g1}} = \phi + \theta_{\text{g1}}, \quad \theta_{\text{g114}} = \phi + \theta_{\text{g1}} + \theta_{\text{a4}}
\]

Where, \( l_{46} = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2} \)

\[
l_{46} = \sqrt{(x_4 - x_1)^2 + (y_4 - y_1)^2}
\]

Here,

\[
\theta_{\alpha_4} = \theta_{\text{a1}} + \delta_1, \quad \theta_{\text{a4}} = \theta_{\text{d1}} + \delta_2, \quad \delta_1 = \theta_{\text{c1}}, \quad \delta_2 = 2\theta_{\text{d1}} = 2\theta_1
\]

The gains required to model Jacobian can be evaluated
by differentiating Equations 26 and 27 with respect to time as,
\[
\begin{bmatrix}
\dot{X}_{ap} \\
\dot{Y}_{ap}
\end{bmatrix}
= \begin{bmatrix}
\dot{X}_{CM} - r_3 \phi \dot{\phi} \\
\dot{Y}_{CM} + r_3 \phi \dot{\phi}
\end{bmatrix}
+ \begin{bmatrix}
-l_{1c}(\theta_{phi}) - l_{1s}(\theta_{phi}) & -l_{2c}(\theta_{phi}) - l_{2s}(\theta_{phi}) \\
-l_{3c}(\theta_{phi}) - l_{3s}(\theta_{phi}) & -l_{4c}(\theta_{phi}) - l_{4s}(\theta_{phi})
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix}
+ \begin{bmatrix}
-l_{1c}(\theta_{phi}) - l_{1s}(\theta_{phi}) \\
-l_{3c}(\theta_{phi}) - l_{3s}(\theta_{phi})
\end{bmatrix}\dot{\phi}
\] (28)

From Equation 28, the gains are found from 2nd and 3rd terms as (as shown in Fig. 2): From 2nd term: \(K_1 = a_{12}, K_2 = a_{22}, K_3 = a_{11}, K_4 = a_{21}\) and from 3rd term: \(K_5 = a_{11}, K_6 = a_{21}\). Where, \(a_i\) are the elements of the matrix.

**IV. Simulation and Animation Results**

The proposed control strategy has been validated through simulation and animation results. The simulation is carried out for 25 seconds in all cases as mentioned in Example 1 and 2. However, for Bezier curve tip trajectory as mentioned below, it is carried out for a second. The simulation study deals with four different cases, are discussed as follows:

**A. Joint Trajectory of 6 DOF Planar Space Robot**

In the case of joint trajectory (cases A and B), the desired joint angle for the actuated joint (\(\theta_i\)) in Case A is \(\theta_{sd} = 75^\circ = 1.309 \text{ rad}\) and while in Case B, \(\theta_{sd} = 50^\circ = 0.8727 \text{ rad}\) for two actuated joints (\(\theta_1\) and \(\theta_2\)).

**B. Tip Trajectory of 6 DOF Planar Space Robot**

In the case of tip trajectory (cases C and D), the reference input trajectory to the tip is required. To validate the proposed control strategy, we consider two types of tip trajectories. First one is a circle, whereas other one is a Bezier curve.

**B1. Circular Reference Trajectory**

Let us consider the circular reference trajectory for the 6 DOF planar space robot. The reference trajectory can be expressed as,
\[
\begin{align*}
X_{ap} &= Acos(\psi(t)) + X_0 \\
Y_{ap} &= Asin(\psi(t)) + Y_0
\end{align*}
\] (29)
\[
\begin{align*}
\dot{X}_{ap} &= -A[6(\psi_f - \psi_0)u(t) + \dot{t}_f - 6(\psi_f - \psi_0)u(t) + \dot{t}_f] \sin(\psi(t)) \\
\dot{Y}_{ap} &= A[6(\psi_f - \psi_0)u(t) + \dot{t}_f - 6(\psi_f - \psi_0)u(t) + \dot{t}_f] \cos(\psi(t))
\end{align*}
\] (32)

Further equations (32) and (33) ensure that tip trajectory tracing is traced with zero initial and final velocity.

**B2. Bezier Curve Reference Trajectory**

Let a Bezier curve with four control points (polynomial third degree) is constructed for tip trajectory tracking. The tip displacement and velocity equations are as:
\[
\begin{align*}
X_{ap} &= p_{a0}(1-t)^3 + 3p_{a1}(1-t)^2t + 3p_{a2}(1-t)t^2 + p_{a3}t^3 \\
\dot{X}_{ap} &= -3p_{a0}(1-t)^2 - 6p_{a1}(1-t) + 3p_{a2} + p_{a3}t^2 \\
\ddot{X}_{ap} &= -6p_{a2} + 3p_{a3}t^2 \\
Y_{ap} &= p_{a0}(1-t)^3 + 3p_{a1}(1-t)^2t + 3p_{a2}(1-t)t^2 + p_{a3}t^3 \\
\dot{Y}_{ap} &= -3p_{a0}(1-t)^2 - 6p_{a1}(1-t) + 3p_{a2} + p_{a3}t^2 \\
\ddot{Y}_{ap} &= -6p_{a2} + 3p_{a3}t^2
\end{align*}
\] (33)

Where, \(p_{a0}, p_{a1}, p_{a2}, p_{a3}\) are four control points corresponding to X and Y directions.

The initial configuration of the space manipulator is given by the following joint coordinates in all cases:
\[
\theta = \begin{bmatrix}
0 & 45^\circ & 30^\circ & 30^\circ & 30^\circ & 30^\circ & 30^\circ
\end{bmatrix}
\]

The input parameters used for simulation study are shown in Table 2.

<table>
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<tr>
<th>Robot base and link parameters</th>
<th>(M (\text{kg}))</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotary inertia of base ((I) (\text{kg m}^2))</td>
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<td></td>
</tr>
<tr>
<td>Location of base of arm from vehicle CM ((r) (\text{m}))</td>
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<td></td>
</tr>
<tr>
<td>Length of each link</td>
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<td></td>
</tr>
<tr>
<td>Mass of each link with actuator (kg)</td>
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<td></td>
</tr>
<tr>
<td>Rotary inertia of each link ((\text{kg m}^2))</td>
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</tr>
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</table>

<table>
<thead>
<tr>
<th>Pad parameters</th>
<th>(K_s (\text{Nm/rad}))</th>
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<tbody>
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<td>Stiffness of spring ((K_s) (\text{Nm/rad}))</td>
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<table>
<thead>
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<th>PID Gain parameters</th>
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<th>(K_I)</th>
<th>(K_D)</th>
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<td>Proportional gain</td>
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<td>1000</td>
<td></td>
</tr>
<tr>
<td>Integrative gain</td>
<td>600</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Derivative gain</td>
<td>1</td>
<td>800</td>
<td></td>
</tr>
</tbody>
</table>

| Reference trajectory parameters for cases C and D | \(R_a (\text{m})\) | 0.4 |
|-----------------------------------------------|---------------|
| Radius of circle \((A) (\text{m})\) | 0.8 |
| Initial angle \((\psi_0) = 0^\circ\) | |
| Final angle \((\psi_f = 360^\circ)\) | |
| Final time required to complete \(\psi_f\) \((s)\) | 25 |

**Table 2: Input parameter and its value**

Figure 4 shows the joint trajectory of 6 DOF planar space robot in joint space with a limited number of actuated joints. Figure 4(a) shows joint rotation with respect to time of the robot system with one actuated joint ($\theta_1$). From this figure, it is observed that rotation of $\theta_1$ becomes constant after attaining the desired value of rotation (1.309 rad). Other joint angles have zero rotation as it is locked through PD control law. It is also recognized that the tip of each link fits onto a circular contour. It is further verified by the animation result as shown in Fig. 4(b). And the same control strategy is verified by simulation and animation results as shown in Fig. 4(c) and Fig. 4(d), respectively, with two joint actuation, i.e., $\theta_1$ and $\theta_4$. Here, two backbone curves are used to give the shape to manipulator.

Figure 5 shows the tip trajectory control of 6 DOF planar space robot in the workspace with two and all actuated joints. Figures 5 (a) to (f) are for a given circular tip trajectory reference input whereas Fig. 5 (g) to (k) are for Bezier curve reference input. Figures 5 (a) to (c): Further, Fig. (a), (b) and (c) are for the circular tip trajectory with all actuated joints. Figure 5(a) shows tip trajectory tracking between the reference and the actual command. From this figure, it is evident that the robot tips successfully track the desired trajectory without error. It is also observed that the backbone of manipulator remains onto the circular contour in starting but later it got changed due to all actuated joints. This perfect trajectory tracking is further verified by animation result as shown in Fig. 5(b). Figure 5(c) shows joint rotation with respect to the time of the robot system. From this figure, one can see the rotation of all actuated joints. Figures 5 (d) to (f): Figures 5(d), (e) and (f) are for the circular tip trajectory of robot with two actuated joints. From Fig. 5(d), it is observed that the same trajectory (like Case C) is traced out by the same robot system with only actuations of two joints as shown in Fig. (d). It is further verified by the animation results as shown in Fig. 5(e). The joint rotation shown in Fig. 5(f) illustrates that only joint 2 and 4 contribute to desired tip motion while other joints are kept locked. These locked joints can be utilized as per our requirement such manipulation in the narrow space, avoiding obstacles in the work environment.

Figures 5 (g) to (k): Figure 5(g) shows the Bezier curve tip trajectory of robot with two and all actuated joints. From this figure, it is observed that our proposed 6DOF hyper-redundant space robot gives the exactly same tip trajectory conforming to the reference input irrespective to two/all actuated joints. It is further illustrated by animation results as shown in Figure 5 (h) for all actuated joints and Fig. 5 (i) for two actuated joints. Figure 5 (j) shows the joint rotation of the robot in the case of all actuated joints and Figure 5 (k) shows the joint rotation of the robot in the case of two actuated joints. From figure 5 (k), it is notable that only joints 1 and 4 are kept actuable whereas others are locked. However, the joint actuations
Figure 5. Circular reference input: Tip trajectory of 6 DOF planar space robot, for all actuated joints: (a) simulation result (b) animation result and (c) joint rotation, and for a limited number of actuated joints: (d) simulation result (e) animation result and (f) joint rotation. Bezier curve reference input: 6 DOF planar space robot, (g) simulation result of tip trajectory for two and all actuated joints (h) animation result for all actuated joints (i) animation result for two actuated joints and (j) joint rotation for all actuated joints and (k) joint rotation for two actuated joints.

are not restricted to these joints only. To make robot’s maneuverability more flexible, other joints can also be used for the desired motion planning in narrow and congested spaces. Hence, during motion planning any joints can be utilized as per requirements such as obstacle avoidance in narrow space. And, in a case when robot links come close to obstacles, the corresponding link’s joint can be locked to avoid collision and simultaneously the other locked joint is...
unlocked to keep continue motion planning. The locked joints further can be resumed, if required, in the case of the reverse motion of the link which goes away from the obstacle. Here, it is noticeable that we use only 2DOF at a time out of 6 actuable joints and we ensure that it will be able to provide greater flexibility and maneuverability in small spaces and congested environments.

V. Conclusion and Future Scope
A control strategy for the hyper-redundant (6 DOF) planar space robot has been presented in this paper. For this, the backbone reference set (curve fitting) has been applied for using the redundancy of space robot. For the curve fitting, a circular curve is taken into consideration which controls the link end of the manipulator and hence the whole body of the manipulators. To illustrate the efficacy of the proposed control strategy, four different cases are considered. From these, one can understand that the same motion planning can be achieved by actuation of a limited number of joints what the robot can with all actuated joints. To take care of joint motion and its locking/unlocking the PD control law is used and for tip motion control, the PID controller is used. The efficacy of the proposed control strategy has been successfully verified by simulation and animation results. In future scope, the present work is extended for obstacle avoidance in the workspace and its reconfiguration.

REFERENCE