Topological Structure Coupling-reducing of Parallel Mechanisms

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Abstract: Structure coupling-reducing can reduces the complexity of forward position solutions. But the topic is little being studied so far. This paper studies structure coupling-reducing design methods and novel mechanisms with low coupling-degree. Firstly, a novel concept—structure coupling-reducing (SCR) and motion decoupling (MD) are clarified and revealed. Then, three SCR methods are proposed, which are method based hybrid branched chains, method based superposing kinematic joints, method based separating negative DOFs respectively. At last, SCR examples are illustrated and some novel mechanisms with low coupling-degree are obtained, which offers good potential applications. These SCR methods can be applied to all complex planar mechanisms and spatial mechanisms. They are of great significance in topological structure optimization, kinematics solution, real-time control and application.

Keywords: Parallel mechanisms, Coupling-degree, Structure coupling-reducing, Motion decoupling

0 Introduction

As we all know, forward position solution is one of the most basic important issues in parallel mechanism research, which ultimately attribute to solving a set of nonlinear position equations. Many scholars have contributed a lot to the issue using both algebra method and numerical method. When using the algebra method, by various elimination methods, Grobner bases method and other mathematical methods, the unknowns of position equations are eliminated and ultimately is expressed as solving of a quaternion algebra equation of higher degree\(^{[5]}\). When using numerical method, like chaotic iteration\(^{[6]}\), successive approximation search method\(^{[7]}\), continuous method\(^{[8]}\), single open-chain method\(^{[9]}\) or optimization method such as genetic algorithm\(^{[10]}\), the position equation is transformed into an optimization problem. Two methods require for minimizing the dimensions of nonlinear position equations.

Yang proposed the concept of coupling degree of mechanism in 1983\(^{[11]}\). The coupling degree is used to describe independence among the all loop position variables and can be measured by \(k (k \geq 0)\), which is an important index to describe the complexity of a mechanism topological structure. It also reflects the complexity of kinematics and dynamics solution. \(k\) is constant once the topological structure of a mechanism is determined. It has been proved that the larger the value \(k\) is, the stronger the coupling of loop position variable of the mechanism and the higher of the complexity will be. Further, for the mechanism with \(k=0\), it is an essentially statically determinated structure and the analytical forward position kinematics solution can be easily got. If the mechanism with \(k>0\), its analytical forward position kinematics solution cannot be obtained directly, which needs be solved by simultaneous equations involved multiple loop position variables. The value \(k\) is just as the minimum dimension\(^{[12]}\) of the simultaneous equations.

However, how to reduce the coupling degree \(k\) in order to reduce the dimension of nonlinear position equation of the mechanism? This problem is currently open and not yet studied.

The authors designed 10 weak coupling (i.e., low coupling-degree) novel 6-dof parallel mechanisms\(^{[13, 14]}\) and eight weak coupling three-translational parallel mechanisms\(^{[15]}\) in 2004 and 2005 by using the plane or space hybrid chains including one or two loops. In 2012, by using three kinds of hybrid branches such as simple branches of SPS type, double spherical pair hybrid branches, and triple spherical pair hybrid branches, six kinds of different 6-dof Steward-platform basic models \(1-2-3, 3-1-1-1, 2-2-2, 2-2-1-1, 2-1-1-1-1, 1^6\) are constructed. According to the calculated coupling-degree \(k\) value, all 6-dof Steward parallel mechanisms can be divided into 4 categories, i.e., \(k = 0, 1, 2, 3\). Further, one type with \(k=0\), three types with \(k=1\), one type with \(k=2\) or \(3\) respectively. Then, the numerical forward position solution\(^{[16-18]}\) is obtained according to their coupling degree \(k\) value. However, the general and practical design methods for SCR are not yet presented.

This paper explores an important issues -- how to reduce the coupling-degree of topological structure of a parallel mechanism. We propose three SCR methods, which can be applied to SCR for all complex planar mechanisms and spatial mechanisms.

The rest of this paper is organized as follows. In Section 1, on the basis of defining of new concept of SCR for parallel mechanism, the differences and relationship of the structure coupling-reducing (SCR) and motion decoupling (MD) is revealed. In Section 2, three SCR methods are
proposed by analyzing topological structure of the branched chains and configuration arrangement of the branched chains between the moving platform and the static platform. In Section 3, some novel mechanisms with the coupling-degree being reduced are obtained by using three SCR methods. These novel mechanisms offer good application potential. In Section 4, we give the conclusions.

1 Structure coupling-reducing (SCR) and motion decoupling (MD) of mechanisms

1.1 Structure coupling-reducing (SCR) of a mechanism and its significance

As mentioned above, the inherent association and dependence among each loop position variables, namely high coupling, is one of the most prominent features for a multi-loop spatial parallel mechanism. On the one hand, these characteristics make it have advantages such as strong bearing capacity, small accumulative error, large stiffness etc. On the other hand, configuration design, kinematics analysis and dynamics analysis and the development of control system of the parallel mechanisms have great difficulties, which make a great influence on its application ranges to some extent. Keeping the DOF as well as position and orientation characteristics (POC) of the moving platform remain unchanged, if the coupling-degree can be reduced, not only the complexity of forward position solutions can be directly reduced, but also the calculation of work space, error analysis and control will be easy. Further, that is preferable to make the coupling-degree be \( k = 0 \) because forward position solution of a mechanism with coupling degree \( k = 0 \) can be represented as an analytic expression. The \( k \) value is reduced from a larger one to a small one or even zero, which means to reduce the dimensions of the nonlinear position equations of the mechanism. Therefore, in this paper it is called as "topological structure coupling-reducing (dimension-reducing)"; short for "Structural Coupling-Reducing, SCR". A mechanism could changes from the strong structure coupling to the weak structure coupling by SCR while keeping the DOF and POC remain unchanged. The results bring to advantages to forward position solution or even to obtaining analytical expressions, and the real-time control of kinematics parameters. Besides, it also makes the analysis process of kinematics and dynamics simple. This is the first important issue of topological structure optimization design for mechanisms.

1.2 Motion decoupling (MD) of a mechanism

Motion coupling of a mechanism refers to the correlation and the dependence between the output parameters and the input parameters of a mechanism. Whether linear or nonlinear motion exist this relationship. Therefore, motion decoupling is the key issue of the mechanism parametric design and motion control.

Many researchers have conducted in-depth research of motion decoupling design and analysis for parallel mechanisms. They put forward some parallel mechanisms with partial motion decoupling or full motion decoupling, like the mechanisms with two-rotation \(^{[19,20]}\), one translation and two-rotation\(^{[21]}\), three-translation\(^{[22]}\), 6-dof mechanism\(^{[23]}\), 3-dof mechanism\(^{[24]}\), XYZ parallel elastic mechanisms\(^{[25]}\), 2R-3R-4R spherical\(^{[26]}\) mechanism, 4-dof two-translation and two-rotation parallel mechanisms\(^{[27]}\) a decoupling spherical parallel mechanisms\(^{[28]}\). Tong and Gao analyzed the decoupling characteristics of three forging manipulators by redefining the output parameters and simulation verification\(^{[29]}\). These works focused on the motion decoupling for the special one or a class of parallel mechanisms.

Yang divided the motion decoupling into three types\(^{[30]}\), that is to say, full decoupling, partially decoupling and no decoupling (i.e. strong coupling). The mathematical representation of corresponding input-output elements and three judging rules are also presented, which can guide people to design motion decoupling parallel mechanisms. But the specific design methods are not discussed yet.

Regarding motion decoupling, the authors think that each input parameter should minimize the influence on the number of output parameters or weaken the influence on output parameters. The best situation is that one input variable controls an output variable, which means the input-output motion completely decoupling. For this reason, we call as “input-output motion complete decoupling or partial decoupling” in this paper, short for "Motion Decoupling, MD". Motion decoupling is advantageous to efficient control of position, velocity and acceleration for a mechanism. Therefore, it is the second important issue of topological structure optimization design.

1.3 Differences and relationship between SCR and MD
As described above, SCR and MD of a mechanism are obviously two completely different concepts. The authors have some basic understandings on the differences and relationship between SCR and MD as following.

(1) At present, "decoupling mechanism" that most of the literature referred to is actually MD of mechanisms. Regarding MD, there have being many case studies for both motion decoupling analysis and design at the parallel mechanism community. A general methodology on motion decoupling design needs to be put forward further. On the contrary, there is few research on SCR. The two concepts are easy to confuse, which affects in-depth study of the performance and application of parallel mechanisms.

(2) MD of a mechanism is for the mechanism with more than two DOFs\([30, 31]\). Generally speaking, the more the number of input joint is, the more complex the problem of motion decoupling will be. But MD has nothing to do with the structure coupling-degree \(k\) of the mechanism. For a mechanism with \(k=0\), its input-output relationship can be the strong coupling, and even no MD. While strong structure coupling mechanism with \(k=1, 2, 3\) may have MD, even completely decoupling or partial decoupling.

(3) The measurement of structure coupling degree of a mechanism currently can be described by specific numerical indicators--coupling degree \(k\)[11,12,30]. For the arbitrary planar mechanisms with the number of loops \(v\leq 4\), the coupling degree \(k\) will be only 0, 1, 2 respectively[12]. For any spatial parallel mechanisms, the coupling degree is generally \(k=0, 1, 2, 3\)[18,30,31]. On the contrary, there is no specific numerical indicators to describe the motion decoupling degree currently. So the authors here recommend to use \(Cd\) (de-coupling) to measure motion decoupling degree. The range of \(Cd\) is \(0\leq Cd\leq 1\). When a mechanism with \(Cd=0\), it means complete decoupling. A mechanism with \(Cd = 1\) means not decoupling, i.e. the strong coupling. A mechanism with \(0< Cd< 1\) means partial decoupling, whose details of input-output parameters relationship will depend on the specific topological structure.

(4) Both strong structural coupling and strong motion coupling will effect real-time control of a mechanism. If structural coupling is strong it means the value \(k\) is larger. Then it is difficult to obtain analytical forward position solutions, which will affect the position real-time control. If motion coupling is strong, it is difficult to design control algorithm of the position, velocity and acceleration, which will affect the actual application of parallel mechanisms.

This paper mainly concerns the SCR design methods. Regarding MD will be discussed in another paper.

2 SCR methods for parallel mechanisms

2.1 Coupling degree and a new expression for mechanism composition principle

As what mentioned before, the coupling-degree of mechanisms \(k\) refers to the complexity degree of position variables coupling among each loops in a basic kinematic chain[11,12,31](in short, BKC) what mechanisms contain. This paper gives the definition of coupling-degree according to Ref[11,12] and presents a new analytical expression for mechanism composition principle as follows.

1) The constraint degree of single-open-chain[11,12,30]

The constraint degree \(\Delta_i\) of the \(j^{th}\) SOC\(_j\) in a kinematic chain is defined as

\[
\Delta_j = \sum_{i=1}^{n_j} f_i - I_j - \xi_{L_j} = \begin{cases}
\Delta_j^- = -5, -4, -3, -2, -1, \\
\Delta_j^0 = 0, \\
\Delta_j^+ = +1, +2, +3, L
\end{cases}
\]

(1)

Here, \(m_j\) refers to the number of kinematic pair of the \(j^{th}\) Single-Open-Chain (in short, SOC). \(f_i\) refers to the degree of freedom of \(i^{th}\) kinematic pair(excluding negative degree of freedom); \(I_j\) refers to the number of input pair(s) of the \(j^{th}\) SOC\(_j\); \(\xi_{L_j}\) refers to the number of independent displacement equations of the \(j^{th}\) independent loop.

For a BKC, there must be:

\[
\sum_{j=1}^{v} \Delta_j = 0
\]

(2)

2) Coupling degree of a basic kinematic chain[11,12,30,31]

Coupling degree of a BKC is defined as

\[
k = \frac{1}{2} \min \{ \sum_{j=1}^{v} |\Delta_j| \}
\]

(3)

There may be several structure decomposition schemes when a BKC is decomposed into \(V\) SOC(\(\Delta_j\)), and we should choose the scheme what \(\sum |\Delta_j|\) is the smallest.

Thus, the number of BKC\(_s\) which mechanism contains and its coupling degree can be calculated. The coupling degree of a mechanism \(k\) is the maximum of the coupling degree of BKC\(_s\), which can be expressed by
\[ k = \max \{ k_1, k_2, \ldots, k_n, \ldots \} \quad (4) \]

Here, \( v \) refers to the number of the independent loop, \( k_i \) refers to coupling-degree of the \( i \)th BKC;

3) New expression of a mechanism composition

This paper presents a new accurate expression of mechanism composition, which is shown as follows:

\[ PKM^k = DOF - J_0 + \sum_{j=1}^{v} p_j \cdot BKC^k(\Delta_1, \Delta_2, \ldots, \Delta_i) \quad (5) \]

Here, \( PKM^k \) — the parallel kinematic mechanism with coupling-degree \( k \).

\( DOF - J_0 \) — dof inputs

\( p_j \) — number of BKC with coupling-degree \( k \)

\( BKC^k(\Delta_1, \Delta_2, \ldots, \Delta_i) \) — BKC with coupling degree \( k \), constitute of \( SOC_1, SOC_2, \ldots, SOC_j \) whose constraint degree are \( \Delta_1, \Delta_2, \ldots, \Delta_j \) respectively.

Formula (5) reveals that a parallel kinematic mechanism with coupling-degree \( k \) can be composed of \( dof \) inputs and \( p_j \) BKCs with different coupling-degree like \( BKC^k(\Delta_1, \Delta_2, \ldots, \Delta_i) \), and three information about the topological structure of a mechanism are obtained as follows: ① what are coupling-degree of the mechanism and the number of independent loops? ② what are the number of BKCs and coupling degree of each BKC that the mechanism contains, ③ how is each BKC composed of, including the number of SOC and value of its constraint degree? So the physical meaning of the formula is more clear than that stated in the Ref[11,12,30,31] and suitable for any plane and spatial mechanisms.

2.2 Three SCR design methods

According to Section 0 and Section 1, SCR of a parallel mechanism refers to reducing the coupling degree \( k \) of the mechanism.

So-called the topological structure of a parallel mechanism includes both topological structure of the branched chains and configuration arrangement of the branched chains between the moving platform and static platform. Therefore, we present three SCR design methods from the two aspects as following.

2.2.1 One SCR design method based on the topological structure of branched chain itself

According to the formula (4), the SCR design method based on topological structure of branched chain itself refers that coupling-reducing of each BKC would reduce the coupling-reducing of a mechanism. We can use several BKCs with low coupling-degree to replace a BKC with high coupling-degree, or decompose a BKC with high coupling-degree into several BKCs with low coupling-degree.

Specifically, from this point of view, it is the best to use one or more BKCs with coupling-degree \( k=0 \) as a hybrid chain to replace some branched chain structures. Therefore, we call this method as SCR design method based on hybrid chain.

2.2.2 Two SCR design methods based on configuration arrangement of the branched chains between the moving platform and the static platform.

According to the formula (1) and (3), the value of coupling-degree \( k \) of a mechanism depends on the value of the constraint degree \( \Delta_j \) of the first SOC. So we must reduce the constraint degree \( \Delta_j \) of the first SOC in order to reduce the coupling-degree \( k \) of the mechanism. Therefore, we propose another two SCR methods further as follows.

① SCR design method based on overlapping kinematic joints.

That is to say, we may merge kinematic joints or reduce the number of sides of the moving platform to make the number of DOF in the first SOC decrease according to formula (1), which leads to reduce the constraint degree \( \Delta_j \) of the first SOC.

② SCR design method based on separating negative DOF.

That is to say, we may co-line or overlap the axis of the kinematic joint (include multi-DOF joints, such as spherical pair S, cylindrical pair C, Cardan joint T) in the first SOC, to make one of the DOFs become a negative DOF which is not counted in calculating constraint degree \( \Delta_1 \) for the first SOC according to formula (1). Thus we can reduce the effective DOF in the first SOC. But the negative DOF should be considered in calculating constraint degree \( \Delta_2 \) for the second SOC. Please to see the details in Section 3.3.
3 Application of three SCR methods

3.1 SCR design method based on hybrid chain

3.1.1 Two steps of the method

SCR design method based on hybrid chain includes two steps:

1) Design BKCs with \( k = 0 \) as hybrid chains.

As we know there is only one kind of planar BKC with \( k = 0 \), namely Assur II group, which consists of two bars and three rotation joints.

There may some spatial BKCs with \( k = 0 \). This paper will propose two kinds of spatial BKC with \( k = 0 \) as following.

① Superposing or overlapping two spherical joints at the end of two S-P-S branches, we get a Hybrid Single-Open-Chain (in short, HSOC) with double spherical joint \( S_{3,4} \), i.e., HSOC\(_1\), as shown in Figure 1(a).

![Figure 1](image1.png)

(a)HSOC\(_1\)  (b)HSOC\(_2\)

Figure 1 Two HSOC\(_1\) built up by BKC with \( k=0 \)

We know that if \( S_1P_1S_3-S_4P_2S_2 \) (the underlined indicates the input joint) is selected as the first SOC\(_1\) before overlapping two spherical joints, the constraint degree \( \Delta_1 \) would be calculated by

\[ \Delta_1 = f_1-I_1-\xi = (3*4+2-4)-2-6 = 2 \]

Here, four negative rotation DOFs around the axis of \( S_1S_2, S_2S_3, S_3S_4, S_4S_1 \) respectively should be excluded.

After overlapping two spherical joints, \( S_1P_1S_3, S_4P_2S_2 \) is taken as the first SOC, the constraint degree \( \Delta_1 \) would be calculated by

\[ \Delta_1 = f_1-I_1-\xi = (3*3+2-3)-2-6 = 0 \]

At this moment, three negative rotation DOFs around the axis of \( S_1S_2, S_2S_3, S_3S_4 \) respectively should be excluded.

Then, the constraint degree \( \Delta_1 \) of the first SOC\(_1\) reduce from the original value \( \Delta_1=2 \) to new value \( \Delta_1=0 \). Thus, the HSOC\(_1\) itself is just a BKC with \( k=0 \). Actually the enclosed triangle structure constituted by three S pairs or U pairs, or arbitrary combination of three S pairs or U pairs is equivalent to Assur II group constituted by three lower pair, which is a statically determinated structure.

② Superposing or overlapping three S pairs at the end of three S-P-S branches, we get a HSOC with triple spherical joint \( S_{456} \), i.e., HSOC\(_2\), as shown in Figure 1(b).

Calculation of the constraint degree \( \Delta_1 \) of the first SOC\(_1\), \( S_1P_1S_{456}P_2S_2 \), is the same as above, being zero. We selected \( S_1P_2S_{456}R(S_1S_2) \) as the second SOC\(_2\) and get its constraint degree \( \Delta_2 \) by

\[ \Delta_2 = f_2-I_2-\xi = (5+1+1)-1-6 = 0 \]

Here, the negative rotation DOFs around the axis of \( S_3S_{456} \) should be excluded.

Thus, this structure contains two BKCs, i.e., BKC\(_1\) with \( k_1=0 \) and BKC\(_2\) with \( k_2=0 \)

2) Replacing SOC structures using BKCs with \( k=0 \).

Using these HSOC\(_1\), HSOC\(_2\) or HSOC\(_3\) to replace some corresponding SOCs structures in the original mechanism could reduce the coupling-degree of the original mechanism.

Next we will take a \( S+3-SPS \) spherical mechanism as an example to illustrate the SCR design method based on hybrid chains.

3.1.2 Examples--Coupling-reducing for \( S+3-SPS \) spherical mechanism

① The original 3-DOF \( S+3-SPS \) spherical parallel mechanism before coupling-reducing is shown as Figure 2(a). It has three same branched chains like \( SPS \) type. The fourth branched chain has only one spherical pair \( S_{41} \). Obviously, the output motion of the moving platform of the mechanism is three rotation around spherical pair \( S_{41} \).

Below is analysis for coupling-degree of the mechanism as follows.

1) Determine SOC\(_1\) and its constraint degree \( \Delta_1 \)

Choosing SOC\(_1\) has two options:

\[ (1) \quadSOC \{ -S_{11} - P_{12} - S_{13} - S_{23} - P_{22} - S_{21} - \} \]

\[ \Delta_1 = \sum_{i=1}^{n} f_i - I_i - \xi_i = 10 - 2 - 6 = 2 \]

Or

\[ (2) \quadSOC \{ -S_{11} - P_{12} - S_{13} - S_{41} - \} \]

\[ \Delta_1 = \sum_{i=1}^{n} f_i - I_i - \xi_i = 8 - 1 - 6 = 1 \]

So we choose option (II) which constraint degree \( \Delta_1 \) is
smaller than that of the option (I). Therefore, 
\[ SOC\{-S_{11} - P_{12} - S_{13} - S_{41} -\} \] is selected as the first SOC.

2) Determine SOC2 and its constraint degree \( \Delta_2 \)
\[ SOC_2\{-R(S_1 - S_{41}) - S_{23} - P_{22} - S_{21} -\} \]
\[ \Delta_2 = \sum_{i,j}^m f_i \cdot I_j \cdot \xi_i = 7 - 1 - 6 = 0 \]

3) Determine SOC3 and its constraint degree \( \Delta_3 \)
\[ SOC_3\{-S_{11} - P_{12} - S_{13} - S_{33} -\} \]
\[ \Delta_3 = \sum_{i,j}^m f_i \cdot I_j \cdot \xi_i = 6 - 1 - 6 = -1 \]

4) Determine the BKC which the mechanism contains and its coupling-degree \( k \)
\[ k = \frac{1}{2} \sum_{j=1}^3 |\Delta_j| = \frac{1}{2} (1 + 0 + 1) = 1 \]

So this mechanism has one BKC with \( k = 1 \) and can be expressed as 
\[ PKM_1 = 3-J_n^a + 3BKC^1 (1,0,-1) \]

The coupling degree of the mechanism is 1.

2) Coupling-reducing design for the mechanism
To reduce the structure coupling-degree of the mechanism, we use only one HSOC \( i_1 \) to replace two S-P-S branched chains and get the derivative mechanism as shown in Figure 2(b). Now we give a coupling-reducing analysis for the mechanism below.

1) Determine SOC1 and its constraint degree \( \Delta_1 \)
SOC1 has two options:
\[ (I) \ SOC\{-S_{11} - P_{12} - S_{13} - S_{41} -\} \]
\[ \Delta_1 = \sum_{i,j}^m f_i \cdot I_j \cdot \xi_i = 8 - 2 - 6 = 0 \]
\[ (II) \ SOC\{-S_{11} - P_{12} - S_{13} - S_{41} -\} \]
\[ \Delta_1 = \sum_{i,j}^m f_i \cdot I_j \cdot \xi_i = 8 - 1 - 6 = 1 \]

Obviously, the constraint degree \( \Delta_1 \) of the SOC1 in option (I) is zero and the SOC1 itself is a BKC with \( k = 0 \), so we choose \(-S_{11} - P_{12} - S_{13} - S_{22} -\) as the first SOC.

2) Determine SOC2 and its constraint degree \( \Delta_2 \)
\[ SOC_2\{-R(S_1 - S_{41}) - S_{23} - P_{22} - S_{21} -\} \]
\[ \Delta_2 = \sum_{i,j}^m f_i \cdot I_j \cdot \xi_i = 6 - 0 - 6 = 0 \]

3) Determine SOC3 and its constraint degree \( \Delta_3 \)
\[ SOC_3\{-S_{11} - P_{12} - S_{13} - S_{33} -\} \]
\[ \Delta_3 = \sum_{i,j}^m f_i \cdot I_j \cdot \xi_i = 7 - 1 - 6 = 0 \]

4) Determine the BKC and its coupling degree \( k \)
\[ k = \frac{1}{2} \sum_{j=1}^3 |\Delta_j| = 0 \]

The mechanism has three BKCs with \( k = 0 \) and can be expressed as 
\[ PKM_0^3 = 3-J_n^a + 3BKC^0 (0) \]

The coupling degree of the mechanism is \( k = 0 \). Because the coupling-degree value is reduced from 1 to 0, we could get analytic forward position solution directly. At the same time, degree of freedom (DOF=3) and POC (three-rotation) of the mechanism keep no change.

3.2 SCR design method based on superposing joints
3.2.1 The coupling degree calculation before SCR
Typical 3-RRR planar parallel mechanism is shown in Figure 3(a). The coupling-degree calculation of the mechanism is relatively easy as follows.

We could choose any two chains to constitute the first SOC1 because of three same chains being arranged symmetrically.
1) Determine $SOC_1$ and its constraint degree $\Delta_1$

$$SOC_1 \{-R_{21} \cdot R_{22} \cdot R_{23} \cdot R_{33} \cdot R_{32} \cdot R_{31}\}$$

So, $\Delta_1 = f_1 - I_1 - \zeta_1 = 6 \cdot 2 - 3 = 1$

2) Determine $SOC_2$ and its constraint degree $\Delta_2$

$$SOC_2 \{-R_{11} \cdot R_{12} \cdot R_{13}\}$$

So, $\Delta_2 = f_2 - I_2 - \zeta_2 = 3 \cdot 1 - 3 = 1$

3) Determine the BKC and its coupling-degree $k$:

$$k = 1 = (1 + 1 + 1) = 1$$

The mechanism has one BKC with $k=1$ and can be expressed as

$$PKM_1 = 3-J_{in} + BKC_1 (-1,1)$$

The coupling-degree of the mechanism is $k=1$

3.2.2 Coupling-reducing for 3-RRR mechanism

Now we superpose or overlap any two revolute pairs in the moving platform and get the derived mechanism with low coupling-degree as shown in Figure 3(b). Similarly, we have the following analysis.

1) Determine $SOC_1$ and its constraint degree $\Delta_1$

$$SOC_1 \{-R_{21} \cdot R_{22} \cdot R_{23} \cdot R_{32} \cdot R_{31}\}$$

So, $\Delta_1 = f_1 - I_1 - \zeta_1 = 5 \cdot 2 - 3 = 0$

2) Determine $SOC_2$ and its constraint degree $\Delta_2$

$$SOC_2 \{-R_{11} \cdot R_{12} \cdot R_{13} \cdot R_{33}\}$$

So, $\Delta_2 = f_2 - I_2 - \zeta_2 = 4 \cdot 1 - 3 = 0$

Therefore, this mechanism contains two BKCs, i.e., BKC_1 with $k_1 = 0$ and BKC_2 with $k_2 = 0$. It can be expressed as

$$PKM_0 = 3-J_{in} + 2BKC_0 (0)$$

At this moment, coupling-degree of the mechanism is 0

3.3 SCR method based on separating negative DOFs

A typical configuration of the 3-DOF 3-RR//R//C three-translational (3T) parallel mechanism is shown in Figure 4[30,31,33]. Three branched chains have the same structure. Its three kinematic pair axis are parallel to each other and can be expressed as:

$$SOC \{-R_i \cdot R_{i+1} / / R_{i+2} / / C_{i+1}\}, i = 1,2,3$$

where, R represents revolute pair. C represents cylindrical pair. Symbol // denotes “parallel”. The axis of the pairs on the moving and static platform overlap with one side of the triangle respectively. To explain the change of constraint degree $\Delta$ and coupling-degree $k$ before and after coupling-reducing, we give a brief calculation of the coupling degree.

3.3.1 The coupling degree calculation before SCR

The coupling-degree before SCR can be calculated below.

1) Calculation of DOF and POC

1) POC set of the end link of any branched chain is:

$$\mathbf{M}_b = \left[ \begin{array}{c} t^i \\ p^i \\ r^i \\ \end{array} \right], i = 1,2,3$$

2) we could choose any two branched chains to constitute the first SOC because three same branched chains are arranged symmetrically between the moving and the static platform. Its number of independent displacement equations $L_b$ is:

$$\dim(\mathbf{M}_b) = \dim(\mathbf{M}_b \cup \mathbf{M}_c) = \dim(\mathbf{r}^i / / \mathbf{R}_{i+1}) + \dim(\mathbf{r}^i / / \mathbf{R}_{i+2})$$

Therefore, this mechanism contains two BKCs, i.e., BKC_1 with $k_1 = 0$ and BKC_2 with $k_2 = 0$. It can be expressed as

$$PKM_0 = 3-J_{in} + 2BKC_0 (0)$$

At this moment, coupling-degree of the mechanism is 0

The derived mechanism can also be viewed to be composed by two Assur II groups. The analytical forward position solution of the mechanism is quite easy to get.

Similarly, the coupling-degree decrease from 1 to 0 but the DOF and POC (two translations and one rotation) of the moving platform of the mechanism is still unchanged.

Actually, design of hybrid chains mentioned in section 3.1 can be regarded as basing on superposing kinematic joints. So the SCR method based on hybrid chains can be regarded as a special case of the SCR method based on superposing joints.
\[ \xi_{\Delta_k} = \dim(M_{\text{pos}} \cup M_{\alpha}) = \dim\left( \left[ \begin{array}{c} t^0 \\ t^1 \\ t^2 \\ t^3 \\ t^4 \end{array} \right] \cup \left[ \begin{array}{c} t^0 \\ r^1 \ (/ / R_{11}) \\ r^2 \ (/ / R_{12}) \\ r^3 \ (/ / R_{13}) \\ r^4 \ (/ / R_{14}) \end{array} \right] \right) = 4 \]

4) Determining DOF of the mechanism:
\[ F = \sum_{i=1}^{n} f_i - \sum_{j=1}^{m} \xi_{\Delta_k} = 12 - (5 + 4) = 3 \]

5) Determining POC set of the moving platform:
\[ M_{\text{ps}} = M_{\text{pos}} \cap M_{\Delta_k} = \left[ \begin{array}{c} t^0 \\ t^1 \\ t^2 \\ t^3 \end{array} \right] \]

So when we selected \( R_{11}, R_{21}, R_{31} \) as driving pair of the static platform, the moving platform of the mechanism can realize three-translation movement.

(2) Calculation of coupling degree \( k \)
1) Determine the first SOC1 and its constraint degree \( \Delta_1 \):
\[ SOC\{ - R_{11} / / R_{12} / / C_{13} - C_{23} / / R_{22} / / R_{21} - \} \]
\[ \Delta_1 = \sum_{i=1}^{n} f_i - R_{11} \cdot \xi_{\Delta_k} = 8 - 2 - 5 = 6 \]
2) Determine SOC2 and its constraint degree \( \Delta_2 \):
\[ SOC\{ - R_{31} / / R_{32} / / C_{33} - \} \]
\[ \Delta_2 = \sum_{i=1}^{n} f_i - R_{31} \cdot \xi_{\Delta_k} = 4 - 1 = 3 \]
3) Determine the BKC and its coupling degree \( k \):
\[ \kappa = \frac{1}{2} \sum_{i=1}^{2} |\Delta_i| \cdot \frac{1}{2}(|t^1| + |t^2|) = 1 \]

So the mechanism contains one BKC with \( k = 1 \) and can be expressed as
\[ PKM^1 = 3 - J_{\Delta_k} + BKCl^1(1, -1) \]
The coupling degree of the mechanism is \( k = 1 \). Therefore, its numerical forward position solutions can be get by one-dimensional search method.

3.3.2 Coupling-reducing for three-translation mechanism
According to SCR method based on separating negative DOFs stated in the Section 2.2.2, we need to parallel or superpose the axis of two revolute pair (or that of cylindrical pair) of any two branched chains and make these two branched chains constitute the first loop. The operation will reduce the constraint degree \( \Delta_1 \) of the first loop. For that we propose two coupling-reducing configurations. The first configuration is to parallel the axis of revolute pair R1 of the first and second chain in the static platform, as shown in Figure 5(a). The second configuration is to superpose the axis of revolute pair R of the first and second chain in the static platform as shown in Figure 5(b).

(a) Coupling-reducing (1) (b) Coupling-reducing (2)

\[ \text{Figure 5 Two SCR configurations of 3T Mechanism} \]

(1) Calculation of DOF
1) We choose the first and the second branched chain with parallel axis of revolute pair R in the static platform to constitute the first loop. The number of independent displacement equations \( \xi_{\Delta_k} \) is:
\[ \xi_{\Delta_k} = \dim(M_{\text{pos}} \cup M_{\alpha}) = \dim\left( \left[ \begin{array}{c} t^0 \\ r^1 \ (/ / R_{11}) \\ r^2 \ (/ / R_{12}) \end{array} \right] \right) = 4 \]

2) DOF and POC of the sub-parallel mechanism made up by the two branched chains are:
\[ F = \sum_{i=1}^{n} f_i - \sum_{j=1}^{m} \xi_{\Delta_k} = 8 - 4 = 4 \]

\[ M_{\text{ps(1-2)}} = \left[ \begin{array}{c} t^0 \\ t^1 \\ t^2 \end{array} \right] \]

3) Determining the number of independent displacement equations of the second SOC \( \xi_{\Delta_k} \):
\[ \xi_{\Delta_k} = \dim(M_{\text{pos}} \cup M_{\alpha}) = \dim\left( \left[ \begin{array}{c} t^0 \\ r^1 \ (/ / R_{11}) \\ r^2 \ (/ / R_{12}) \end{array} \right] \right) = 4 \]

4) Determining DOF of the mechanism:
\[ F = \sum_{i=1}^{n} f_i - \sum_{j=1}^{m} \xi_{\Delta_k} = 12 - (4 + 5) = 3 \]

5) Determining POC set of the moving platform:
\[ M_{\text{ps}} = M_{\text{pos}} \cap M_{\Delta_k} = \left[ \begin{array}{c} t^0 \\ t^1 \\ t^2 \end{array} \right] \]

Before and after coupling-reducing, the number of independent displacement equations of the first and second SOC have changed, but DOF and POC stay the same.

(2) Calculation of coupling degree \( k \)
1) Determine SOC1 and its constraint degree \( \Delta_1 \):
\[ SOC\{ - R_{11} / / R_{12} / / C_{13} - C_{23} / / R_{22} / / R_{21} - \} \]
\[ \Delta_1 = \sum_{i=1}^{n} f_i - R_{11} \cdot \xi_{\Delta_k} = 6 - 2 - 4 = 0 \]
It is easy to judge from that the translational DOF \( P^{(C13)} \) along the axis of \( C_{13} \) and translation DOF \( P^{(C23)} \) along the axis of \( C_{13} \) are negative DOF. So they should not be counted in the calculation of constraint degree \( \Delta_1 \). But it
should be considered into the calculation of $\Delta_2$ of SOC2 because it is an effective DOF for SOC2.  
2) Determine SOC2 and its constraint degree $\Delta_2$

$$SOC\{ -P^{(C_{13})} - P^{(C_{23})} - R_{31} / \{C_{33} \} \}$$

$$\Delta_2=\sum_{i=1}^{m_0} f_i \Delta r_i - \sum_{i=1}^{n_0} \Delta \xi_i = 6 - 1 - 5 = 0$$

($P^{(C_{13})}$, $P^{(C_{23})}$ DOF of SOC1 has been included in the calculation of $\Delta_2$.)

3) Determine the BKC and the coupling degree $k$

$$k = \frac{1}{2} \sum_{i=1}^{m_0} |A_i| = 0$$

Thus the mechanism contains two BKCs with $k=0$ and can be expressed as

$$PKM^0 = 3 - J_{\theta} + 2BKC^0 (0).$$

The coupling degree of the mechanism is $k=0$

We can also calculate the coupling-degree of another special configurations shown in Figure 5(b) being with $k=0$, which indicate that the structure coupling-degree of the configuration have been reduced to zero. Its analytic forward position solutions is also easy to be obtained.

4 Conclusions

This paper focuses on the topological structure coupling-reducing of the parallel mechanisms. Firstly, as a novel concept--SCR of a parallel mechanism is proposed and defined. The differences and relationship of SCR and MD are also clarified and revealed. Keeping DOF and POC remain invariable, from the point of view of topological structure of the branched chains and configuration arrangement of the branched chains between the moving platform and static platform, three SCR method, i.e., SCR method based on hybrid chain, SCR method based on superposing joints, SCR method based on removing negative DOFs, are proposed respectively. And SCR examples corresponding to the three SCR methods are illustrated respectively. Meanwhile some novel mechanisms with low or zero coupling degrees are obtained, which offer potential application of these optimizing configurations.

This paper also gives the new expression for the mechanism composition which reveal the composition principle more clearly.

Three SCR methods can be applied to all complex planar mechanisms and spatial mechanisms. They are of significance in topological structure optimization design, kinematics solution, real-time control and application.

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