Heat Conduction Model of TBM Disc Cutter Cutting Temperature and Its Solution

Liwei Song1
Department of mechanical engineering, college of Mechanical Engineering, Tianjin University, Tianjin, China

Wei Guo2
Department of mechanical engineering, college of Mechanical Engineering, Tianjin University, Tianjin, China

Dianhua Zhu3*
Department of mechanical engineering, college of Mechanical Engineering, Tianjin University, Tianjin, China

Abstract: Disc cutter is Full face rock tunnel boring machine (TBM) important rock breaking tool, cutting heat generation has an important influence on cutter life, disc cutter cutting temperature characteristic research be imperative. Based on heat conduction theory to establish disc cutter cutting temperature heat conduction model, and derive its solution; analyze influence on disc cutter cutting temperature factors, the disc cutter cutting temperature follow penetration and disc cutter velocity and rock strength increases to rise, follow the time factor increases to rise until stabilization. By TB880E cutterhead as example to research disc cutter cutting temperature distribution characteristics on the cutterhead, the more closed to the cutterhead edge, the disc cutter cutting temperature is higher, it’s the certain related to the mounting radius and the cutter spacing, and have a certain guiding significance for the cooling pipes reasonable arrangement and disc cutter layout design on the cutterhead.

Keywords: Heat conduction theory, Disc cutter cutting temperature, Excavation parameters, Time and geological factor

I. Introduction

In the cutter cutting process, due to the friction effect between cutter and cutting the object make the cutter temperature rise, from the temperature change in the metal cutter cutting process can be seen, heat generation and transmission problems of metal cutter cutting work piece has an important influence on cutter life and cutting force, therefore, research on cutting temperature influence on cutter in cutting process has become an important topic [1]. With the wide application of full face rock tunnel boring machine in tunnel engineering, disc cutter as the main cutting tools, under semi closed environment conditions to cut rock, because of friction and environmental factors make disc cutter cutting temperature rise and not easy to reduce, to speed up the disc cutter wear, has a great impact on its service life, its cutting force distribution and excavation efficiency, will cause serious consequences and even cutterhead failure [2-4], therefore, research on disc cutter cutting temperature distribution characteristics on the cutterhead, to guide the cooling pipes arrangement and disc cutter layout optimization design has a certain engineering significance.

Cutting temperature research method is divided into: Mathematical analytical method [5], Test method, Numerical method [6], Mixed method, Heat source method [7-8].

II. Disc cutter cutting temperature characteristics research

In the TBM excavation process, disc cutter cutting rock, contact and friction with rock, from the first law of thermodynamics, part of disc cutter work on rock will be used to change internal energy, cause temperature rise, high temperature can make rock strength reduced, conducive to rock crushed, improve excavation efficiency, but high temperature will make the disc cutter material performance reduced, accelerate disc cutter wear, not conducive to disc cutter rock breaking excavation, therefore, reasonable control of temperature is necessary. Fig.1 shows disc cutter cutting temperature during the rock breaking changed regional.
A. Disc cutter cutting temperature three-dimensional heat conduction model establish

From disc cutter temperature field change regional can be seen, there is 3 surfaces contact with the rock in the disc cutter cutting rock process, create friction heating phenomenon, respectively are disc cutter blade width and two sides, cutterhead rotation velocity \( \omega_d \) drive disc cutter rotation speed \( \omega \), and every revolution penetration as \( h \) velocity along the axial direction to excavate, make disc cutter cutting rock. Assuming the disc cutter is V-type tool, and homogeneous material, could the disc cutter model is simplified as a cylindrical model \( T(r, \varphi, z, t) \). Disc cutter surface and zero space convection heat transfer, the initial temperature is zero degrees, when \( t>0 \), due to friction effect in the cutting process produced heat source intensity is \( q \), cutting edge angle is \( \psi \), make disc cutter center as fixed coordinate system, Fig.2 shows disc cutter in the cutting rock process geometric dimensions and boundary conditions, in cylindrical coordinates the three-dimensional heat conduction problem mathematical expression as:

\[
\begin{align*}
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\omega_t}{\alpha} \frac{\partial T}{\partial \varphi} + \frac{1}{\rho c_p} \frac{\partial T}{\partial t} &= \frac{1}{\alpha} \frac{\partial T}{\partial t} {\quad (1)} \\
0 &\leq r \leq b, 0 \leq \varphi \leq 2\pi, -h \leq h \tan \psi \leq z \leq h \tan \psi, t > 0 \quad \text{(2)} \\
B.C. -k \frac{\partial T}{\partial r} + HT &= 0 r = 0, 0 \leq \varphi \leq 2\pi, t > 0 \quad \text{(3)} \\
I.C. T &= 0 t = 0 \quad \text{(4)} \\
g(r, \varphi, z, t) = \begin{cases} 
\frac{\Theta_0}{2\pi} \delta(r) 0 \leq r \leq b, 0 \leq \varphi \leq 2\pi, -h \tan \psi \leq z \leq h \tan \psi \\
0 &\text{Other areas}
\end{cases} \quad \text{(5)}
\]

Where, \( H \) —boundary and the environment heat transfer coefficient, \( k \) —medium material thermal conductivity coefficient, \( \rho, c_p \) —Microelement density and specific heat capacity at constant pressure, \( \alpha = \frac{k}{\rho c_p} \) —Thermal diffusion coefficient or thermal conductivity coefficient.

Order \( T = \Theta \exp \left(-\frac{\omega_t \varphi - \omega_\varphi^2}{4\alpha t}\right) \) {\quad (6)}

\[
\begin{align*}
\frac{d^2 \Theta}{dr^2} + \frac{1}{r^2} \frac{d^2 \Theta}{d\varphi^2} + \frac{\omega_t}{\alpha} \frac{d\Theta}{d\varphi} + \frac{1}{\rho c_p} \frac{d\Theta}{dt} &= \frac{1}{\alpha} \frac{d\Theta}{dt} {\quad (7)} \\
0 &\leq r \leq b, 0 \leq \varphi \leq 2\pi, -h \leq h \tan \psi \leq z \leq h \tan \psi, t > 0 \quad \text{(8)} \\
B.C. -k \frac{\partial \Theta}{\partial r} + HT &= 0 r = 0, 0 \leq \varphi \leq 2\pi, t > 0 \quad \text{(9)} \\
I.C. \Theta &= 0 t = 0
\end{align*}
\]

B. Disc cutter cutting temperature model solution

According to the disc cutter cutting temperature heat conduction model, this problem three dimensional Green function can be composed by three one-dimensional Green function, so the model used for analytical analysis, acquirable:

\[
G(r, \varphi, z, t|\varphi', z', t') = G_1(r, \varphi', t'|\varphi, z, t)G_2(\varphi, \psi', t|\varphi', z')G_3(z, \psi, t) \quad \text{(10)}
\]

According to the boundary condition formula (7) and \( 0 \leq r \leq b \) regional range, the proper function and the modulus can be respectively:

\[
\beta_m R_v(r) = J_v(\beta_m r) \quad \text{(11)}
\]

Proper value \( \beta_m \) to satisfy the positive root of the following equation:

\[
\beta_m J_v'(\beta_m b) + H \frac{J_v(\beta_m b)}{\beta_m^2} = 0 \quad \text{(12)}
\]

\[
J_v(\beta_m) = \sum_{n=0}^{\infty} \left(\frac{(-1)^n}{n! (v+n+1)} \left(\frac{\beta_m b}{2}\right)^{2n+v}\right) \quad \text{(13)}
\]

The Green function can be obtained:

\[
G_1(r, \varphi', t'|\varphi, z, t) = \sum_{m=1}^{\infty} \frac{2\beta_m^2 J_v(\beta_m r) J_v(\beta_m r')}{J_v(\beta_m b)(\beta_m^2 r^2 + \beta_m^2 b^2 - 2r b \cos \psi)} \exp[-a \beta_m (t - t')] \quad \text{(14)}
\]

At the same time, due to the disc cutter geometric parameters symmetrical characteristic \( \nu = 0 \), \( G_1(r, \varphi', t') \) can be simplified as:

\[
G_1(r, \varphi', t') = \sum_{m=1}^{\infty} \frac{2\beta_m^2 J_v(\beta_m r) J_v(\beta_m r')}{J_v(\beta_m b)(\beta_m^2 r^2 + \beta_m^2 b^2 - 2r b \cos \psi)} \exp[-a \beta_m (t - t')] \quad \text{(15)}
\]

Due to the disc cutter approximation for the entire circular column heat conduction problem, according to the boundary condition and \( 0 \leq \varphi \leq 2\pi \) regional range, the proper function as:

\[
\phi(\nu, \varphi) = \sum_{n=0}^{\infty} \left(A_n \sin \nu \varphi + B_n \cos \nu \varphi\right) \quad \text{(16)}
\]

Using the orthogonality of trigonometric function expanse coefficient:

\[
A_n = \left\{ \begin{array}{ll}
\frac{1}{\pi} \int_0^{2\pi} \phi(\nu, \varphi) \sin \nu \varphi d\varphi & \nu = 1,2,3,\ldots \\
0 & \nu = 0
\end{array} \right. \quad \text{(17a)}
\]

\[
B_n = \left\{ \begin{array}{ll}
\frac{1}{\pi} \int_0^{2\pi} \phi(\nu, \varphi) \cos \nu \varphi d\varphi & \nu = 1,2,3,\ldots \\
0 & \nu = 0
\end{array} \right. \quad \text{(17b)}
\]

The formula (17) brought into formula (16) can be obtained:

\[
\phi(\nu, \varphi) = \sum_{n=0}^{\infty} \int_0^{2\pi} \phi(\nu, \varphi') \cos \nu (\varphi - \varphi') d\varphi' \quad \text{(18)}
\]

\[
G_2(\varphi, \psi, t) = \sum_{n=0}^{\infty} \cos \nu (\varphi - \psi) \quad \text{(19)}
\]

(3) \( G_3(z, \psi, t) \) solution:

According to the solution properties of orthogonal coordinates:
\[
G_3(z, t; x, t) = \frac{1}{\sqrt{4\pi a(t-r)}} \exp \left(\frac{-(x-y)^2}{4a(t-r)}\right)
\]

(20)

Finally can be obtained:
\[
G(r, \varphi, z, t; x, \varphi, z', t) = \frac{a}{k} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{2\beta E_r}{b^2 \chi \beta (\beta_m \beta_n)} \exp[-a \beta_m(t-\tau)],
\]

\[
\frac{1}{\pi} \sum_{n=0}^{\infty} \phi(\varphi, \varphi') \cos(\varphi - \varphi') \exp \left[\frac{-(x-y)^2}{4a(t-r)}\right]
\]

(21)

Therefore,
\[
\theta(r, \varphi, z, t) = \int_0^t dt \int_0^1 dz \int_{-\tan \psi}^{\tan \psi} \frac{q_0}{2\pi r^2} \delta(r')
\]

(22)

\[
T = \theta(2a) \left(\frac{\alpha r^2}{2a} - \frac{\alpha^2}{4a t}\right)
\]

(23)

\[
W_\mu = \mu F L = \mu C \frac{aWh}{1 + \gamma} \left(\frac{S_\alpha^2}{\gamma_{Wh}}\right)^{1/2} L
\]

(24)

Where, \(W_\mu\) is the disc cutter cutting path consumed friction energy, \(F\) is the single disc cutter cutting rock resultant force, \(\varphi\) is the disc cutter cutting rock contact angle, \(W\) is the disc cutter edge width, \(b\) is the disc cutter radius, \(\sigma_t\) is the rock uniaxial compressive strength, \(\gamma\) is the rock tensile strength, \(h\) is the penetration, \(Y\) Is the cutting edge pressure distribution coefficient (if disc cutter is V-type cutter, \(Y = 0.2\), disc cutter edge width is large, \(Y = -0.2\), commonly \(Y = 0.1\), \(C\) is a dimensionless coefficient, \(C \approx 2.12\), \(\mu\) is the friction coefficient, \(L\) is the disc cutter linear breaking rock path, \(L = \omega b t\).

Heat source intensity \(q_0\) is in unit time through a unit area the quantity of heat transfer, i.e:
\[
q_0 = \frac{W_\mu}{2\pi b (2h \tan \psi + W)}
\]

(25)

Where, \(S_r\) is reference heat transfer surface section's acreage.

The formula (25) brought into formula (23), finally obtain the disc cutter cutting temperature changing formula:
\[
T = \frac{Haq_0W_\mu}{\pi^2 k (2h \tan \psi + W)(1 + \gamma)} \left[\frac{S_\alpha^2}{\gamma_{Wh}}\right]^{1/2} \int_0^1 \frac{1}{\sum_{m=1}^{\infty} \frac{\alpha h (\beta_m \beta_n \exp[-a \beta_m(t-\tau)])}{b \alpha (\beta_m \beta_n h^2 + b \alpha^2)}} \int \frac{1}{\exp(\frac{z+\tan \psi}{4a(t-\tau)}) - \exp(-\frac{z+\tan \psi}{4a(t-\tau)})} \frac{dz}{dz} \int_0^t dt
\]

(26)

When \(r = b, z = 0\) obtained the disc cutter highest temperature model:
\[
T = \frac{2Haq_0W_\mu}{\pi^2 k (2h \tan \psi + W)(1 + \gamma)} \left[\frac{S_\alpha^2}{\gamma_{Wh}}\right]^{1/2} \int_0^1 \frac{1}{\sum_{m=1}^{\infty} \frac{\alpha h (\beta_m \beta_n \exp[-a \beta_m(t-\tau)])}{b \alpha (\beta_m \beta_n h^2 + b \alpha^2)}} \frac{dz}{dz} \int_0^t dt
\]

(27)

From analytic method obtained disc cutter cutting temperature model can be seen, excavation parameter (penetration and disc cutter cutting velocity) and time and geological factors have influence on the disc cutter cutting temperature, then select different penetration and disc cutter cutting velocity and time factor under the different geological conditions to analyze disc cutter cutting temperature, disc cutter parameters as shown in TABLE I, rock parameters as shown in TABLE II.

<table>
<thead>
<tr>
<th>Cutter material</th>
<th>Cutter type</th>
<th>Specific heat capacity ((J kg^{-1} K^{-1}))</th>
<th>Thermal conductivity coefficient ((W m^{-1} K^{-1}))</th>
<th>Thermal diffusivity ((m^2 s^{-1}))</th>
<th>Thermal expansivity (\times 10^{-6} K^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>42CrMo</td>
<td>17-inch</td>
<td>461</td>
<td>43.2</td>
<td>3.26</td>
<td>11.1</td>
</tr>
</tbody>
</table>

TABLE I. Disc cutter parameters
A. Excavation parameter (penetration and disc cutter cutting velocity) influence on disc cutter cutting temperature

According to the disc cutter and rock parameters, when the disc cutter cutting velocity is $\omega = 20 \text{rad/min}$, penetration from 0.002m to 0.016m, the relationship between disc cutter cutting temperature and penetration is shown in Fig.3. From Fig.3 can be seen, the disc cutter cutting temperature follow the penetration increases to rise. When disc cutter penetration is 0.01m, disc cutter cutting velocity from 0 to 100 rad/min, the relationship between disc cutter cutting temperature and disc cutter cutting velocity is shown in Fig.4. From Fig.4 can be seen, the disc cutter cutting temperature follow disc cutter cutting velocity increases to rise. Meanwhile, rock strength is stronger, the friction coefficient is large, the disc cutter cutting temperature is higher.

B. Time factor influence on disc cutter cutting temperature

Assuming that other conditions have been given, when penetration is 0.01m and disc cutter cutting velocity is $\omega = 20 \text{rad/min}$, rock breaking time from 1s to 1000s, the relationship between disc cutter cutting temperature and time factor is shown in Fig.5. From Fig.5 can be seen, with disc cutter cutting time increasing, the disc cutter cutting temperature gradually rise, and tends to a stable state. Meanwhile, rock strength is stronger, the friction coefficient is large, the disc cutter cutting temperature is higher.

In summary, disc cutter’s excavation parameter (penetration and disc cutter cutting velocity) and time and geological factors have influence on the disc cutter cutting temperature variation regularity consistent with LuJian Xie [23] master thesis research disc cutter cutting temperature characteristics, this analytical model has some reference value.

IV. Disc cutter cutting temperature on the cutterhead distribution characteristic analysis

Using excavated Peach Blossom Shop No.1 Tunnel, Grinding Groove Ridge Tunnel, Qinling Mountains 1 lines Tunnel exit and so on Tunnel engineering excavation tasks TB880E-8800mm open type hard rock tunnel boring machine cutterhead as an example, research on disc cutter cutting temperature on the cutterhead distribution characteristic under different geological construction conditions. TB880E full face rock tunnel boring machine cutterhead diameter is 8.8m, a total layout of 71 17-inch disc cutter on the cutterhead(disc cutter parameters refer to TABLE 1), including the 6 Center disc cutters, disc cutter spacing is 84mm; the 51 Normal disc cutters, disc cutter spacing is from 65mm to 70mm; the 14 Gauge disc cutters, disc cutter spacing from 65mm gradually reduce to 31mm, the average disc cutters spacing on the cutterhead is 65mm. The drawing of cutters on the TB880E cutterhead is shown in Fig.6, disc cutter spacing position is shown in Fig.7.

### TABLE 1: Rock parameters

<table>
<thead>
<tr>
<th>Thermal conductivity coefficient ($\kappa/\text{Wm}^{-1}\text{K}^{-1}$)</th>
<th>3.55</th>
<th>2.09</th>
<th>2.68</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal diffusivity ($\alpha/\text{mm}^2\cdot\text{s}^{-1}$)</td>
<td>0.0018</td>
<td>0.001</td>
<td>0.0013</td>
</tr>
<tr>
<td>Thermal expansivity ($\times 10^{-3}\text{K}^{-1}$)</td>
<td>1.5</td>
<td>1.5</td>
<td>3</td>
</tr>
<tr>
<td>Friction coefficient (dry)</td>
<td>0.38 ~ 0.42</td>
<td>0.35 ~ 0.40</td>
<td>0.47 ~ 0.55</td>
</tr>
</tbody>
</table>

### Formula Analysis

![Fig. 3. The relationship between disc cutter cutting temperature and penetration](image3)

![Fig. 4. The relationship between disc cutter cutting temperature and disc cutter cutting velocity](image4)

![Fig. 5. The relationship between disc cutter cutting temperature and time factor](image5)

![Fig. 6. The layout drawing of cutters on the TB880E cutterhead](image6)
A. Geological characteristic parameters

Peach Blossom Shop No. 1 Tunnel construction excavation length is 6016m, located in Xi'an to Hefei railway Shanxi province Danfeng county territory; Grinding Groove Ridge Tunnel construction excavation length is 4820m, located in the juncture of Shanxi province Danfeng country and Shangnan county; Qinling Mountains 1 lines Tunnel exit construction excavation length is 5621m, located in Xi'an to Ankang railway Qinling Mountains tunnel, the geological characteristic parameters such as shown in TABLE III.

<table>
<thead>
<tr>
<th>Project name</th>
<th>Peach Blossom Shop No. 1 Tunnel</th>
<th>Grinding Groove Ridge Tunnel</th>
<th>Qinling Mountains 1 lines Tunnel exit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock characteristics</td>
<td>Quartz-schist and Marble</td>
<td>Mica quartz schist</td>
<td>Migmatitic granite Migmatitic gneiss</td>
</tr>
<tr>
<td>The uniaxial compressive strength</td>
<td>70~130Mpa</td>
<td>59~127Mpa</td>
<td>117<del>192Mpa 105</del>325Mpa</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>10~25Mpa</td>
<td>5~20Mpa</td>
<td>7<del>25Mpa 5</del>20Mpa</td>
</tr>
<tr>
<td>Friction coefficient (dry)</td>
<td>0.45~0.48</td>
<td>0.38~0.42</td>
<td>0.47<del>0.55 0.47</del>0.55</td>
</tr>
</tbody>
</table>

TABLE III. Geological characteristic parameters

B. Disc cutter cutting temperature on the cutterhead distribution characteristic

According to the actual construction parameters of TB880E cutterhead, select the highest velocity 5.4rad/min on the cutterhead technical parameters, penetration is 0.01m, Separately to analyze disc cutter cutting temperature on the cutterhead distribution characteristic in the Peach Blossom Shop No.1 Tunnel, Grinding Groove Ridge Tunnel, Qinling Mountains 1 lines Tunnel exit under different geological conditions construction process, as shown in Fig.8 to Fig.10.

From Fig.8 to 10 can be seen, under different geological construction conditions, disc cutter cutting temperature on the TB880E cutterhead follow geological characteristic parameters (the uniaxial compressive strength and tensile strength) and friction coefficient (dry) characteristic.
coefficient (dry) increase to rise, and in the same geological construction conditions, the more closed to the cutterhead edge, the disc cutter cutting temperature is higher, this also corresponds with the view that disc cutter cutting temperature follows the disc cutter cutting velocity increases to rise, also explain the relationship between disc cutter cutting temperature and installation radius and the cutter spacing. This phenomenon is corresponded to cutter temperature follows cutting parameters change to change.

V. Conclusions

In the disc cutter cutting rock process, according to the principle of energy transformation, because of the role of friction resistance, would cause the change of disc cutter temperature, through research on disc cutter cutting temperature characteristic, obtained the following conclusions.

This paper is based on heat conduction theory, to establish disc cutter cutting temperature in rock breaking process three-dimensional heat conduction model, using mathematical analysis method to solve this model. Analyzing excavation parameter (penetration and disc cutter cutting velocity) and time and geological factors influence on the disc cutter cutting temperature, the disc cutter cutting temperature follow excavation parameter (penetration and disc cutter cutting velocity) increases to rise, with disc cutter cutting time increasing, the disc cutter cutting temperature gradually rise, and tends to a stable state. Meanwhile, under different geological construction conditions, rock strength is stronger, the friction coefficient is large, the disc cutter cutting temperature is higher.

Using excavated Peach Blossom Shop No. 1 Tunnel, Grinding Groove Ridge Tunnel, Qinling Mountains I lines Tunnel exit and so on Tunnel engineering excavation tasks TB880E-8800mm open type hard rock tunnel boring machine cutterhead as an example, research on disc cutter on the cutterhead under different installation position and geological construction conditions, coupling rock breaking disc cutter cutting temperature distribution characteristics, as can be seen, disc cutter group cutting temperature on the TB880E cutterhead follow geological characteristic parameters and friction coefficient (dry) increase to rise, and in the same geological construction conditions, the disc cutter cutting temperature follows installation radius and the cutter spacing increase to rise, the more closed to the cutterhead edge, the disc cutter cutting temperature is higher. Through research on the disc cutter cutting temperature distribution characteristics on the cutterhead, could increase the cooling pipes arrangement in closed to edge of cutterhead normal disc cutter region and gauge disc cutter region, at the same time, to guide the disc cutter on the cutterhead layout optimization design has a certain reference value.

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